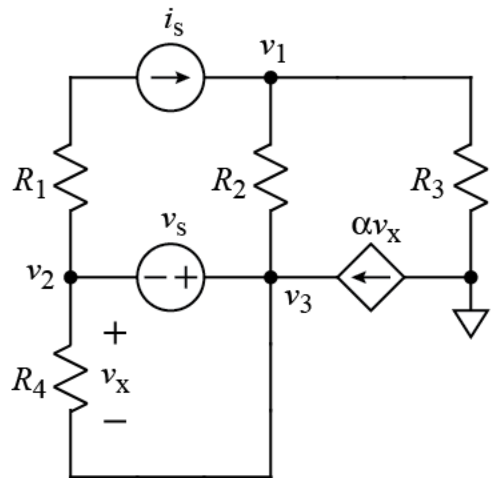
Ex:



a) For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

b) Make a consistency check on your equations for part (a) by setting resistors and sources to numerical values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that your equations for part (a) are satisfied for these values. (In other words, plug the values into your equations for part (a) and show that the left side and the right side of each equation are equal.)

SOL'N: a) First, we define the variable for the dependent source in terms of node voltages.

$$v_x = v_2 - v_3$$

Now we write current sum equations for each node, starting with node v_1.

$$\text{node } v_1: -i_s + \frac{v_1 - v_3}{R_2} + \frac{v_1}{R_3} = 0 \text{ A}$$

We discover that nodes v_2 and v_3 form a supernode, for which we first write a current summation equation.

$$\text{supernode } v_2, v_3: i_s + \frac{v_2 - v_3}{R_4} + \frac{v_3 - v_2}{R_4} + \frac{v_3 - v_1}{R_2} - \alpha(v_2 - v_3) = 0 \text{ A}$$

Our other equation for the supernode is a voltage difference equation.

$$\text{supernode } v_2, v_3: v_s = v_3 - v_2$$

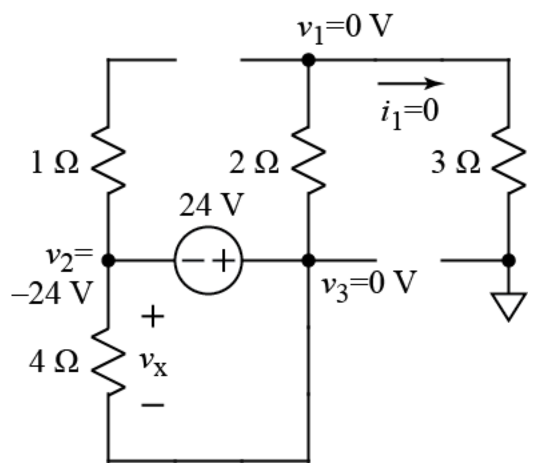
The preceding three equations are the answer to the problem.

b) For the consistency check, we set some R's to zero, making them open circuits, and we set some sources to zero. A voltage source set to zero acts like a wire, and a current source set to zero acts like an open circuit.

Here, we have to be careful to provide paths for current from current sources, unless we set those current sources to zero. One possible consistency check is to set both current sources to zero, avoiding the problem of finding current paths that are different for the two sources. We then pick convenient small integers for other component values:

$$i_s = 0\,\text{A},\ \alpha = 0,\ v_s = 24\,\text{V},\ R_1 = 1\Omega,\ R_2 = 2\,\Omega,\ R_3 = 3\Omega,\ R_4 = 4\,\Omega$$

This yields the following circuit:



Because of the lack of a complete loop for current, we have no current in R_2 and R_3 and no voltage drop for R_2 and R_3. Thus, the voltages at v_1 and v_3 are zero. The v-source then causes the voltage at v_2 to be –24 V.

We now plug the component values and node-voltages into the equations for part (a) to see whether the equations hold.

$$\text{node } v_1 : -i_s + \frac{v_1 - v_3}{R_2} + \frac{v_1}{R_3} = 0\ \text{A}$$

$$\text{node } v_1 : -0\,\text{A} + \frac{0\,\text{V} - 0\,\text{V}}{2\,\Omega} + \frac{0\,\text{V}}{3\,\text{V}} = 0\ \text{A} \quad \text{equation holds} \surd$$

and

$$\text{supernode } v_2, v_3 : i_s + \frac{v_2 - v_3}{R_4} + \frac{v_3 - v_2}{R_4} + \frac{v_3 - v_1}{R_2} - \alpha(v_2 - v_3) = 0 \text{ A}$$

$$\text{supernode } v_2, v_3 : 0\,\text{A} + \frac{-24\,\text{V} - 0\,\text{V}}{4\,\Omega} + \frac{0\,\text{V} - -24\,\text{V}}{4\,\Omega}$$

$$+ \frac{0\,\text{V} - 0\,\text{V}}{2\,\Omega} - 0(-24\,\text{V} - 0\,\text{V}) = 0 \text{ A} \quad \text{equation holds } \surd$$

and

$$\text{supernode } v_2, v_3 : v_s = v_3 - v_2$$

$$\text{supernode } v_2, v_3 : 24\,\text{V} = 0\,\text{V} - -24\,\text{V} \quad \text{equation holds } \surd$$

This consistency check works. Various other checks are possible.