ECE 3510 Lecture 1 notes Introduction to Feedback Systems

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1/10/10
1/6/13

Syllabus.

Supplemental example and problem sessions will make this class much easier.

But it's still a 4-hour engineering class. How can you survive??

1. Easiest way to get through school is to actually learn and try to retain what you are asked to learn.

   Even if you're too busy, don't lose your good study practices.
   What you "just get by" on today will cost you later.

   Don't fall for the "I'll never need to know this" trap. Sure, much of what you learn you may not use, but some you will need, either in the current class, or future classes, or maybe sometime in your career. Don't waste time second-guessing the curriculum, It'll still be easier to just do your best to learn and retain.

2. Don't fall for the "traps".

   Homework answers, Problem session solutions, Posted solutions, Lecture notes.

3. KEEP UP! Use calendar.

4. Make "permanent notes" after you've finished a subject or section and feel that you know it.

Signals (INFORMATION !!)

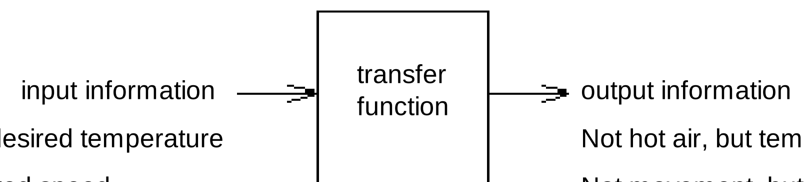
For us: A time-varying voltage or current that carriers information.

Audio, video, position, temperature, digital data, etc...

In some unpredictable fashion
DC is not a signal, Neither is a pure sine wave. If you can predict it, what information is it providing??
Neither DC nor pure sine wave have any "bandwidth".

Recall Fourier series: Any periodic waveform can be represented by a series of sinewaves of different frequencies.

Blocks and block diagrams (acting on signals (information))

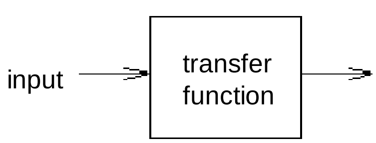


Not cold air, but desired temperature — Not hot air, but temperature of a room

Not fuel, but desired speed — Not movement, but position or speed

These blocks DO NOT show the flow of materials, power, or energy needed to act upon the information. For example, the input to a block might be the postion of the gas pedal in your car and the output might be the car's speed. The energy input required is not shown and niether are the fuel and air moving through the engine. Although they are very important engineering concerns, we will not be considering those things in this class.

If possible we'd like to work with the blocks in a very simple mathmatical way:

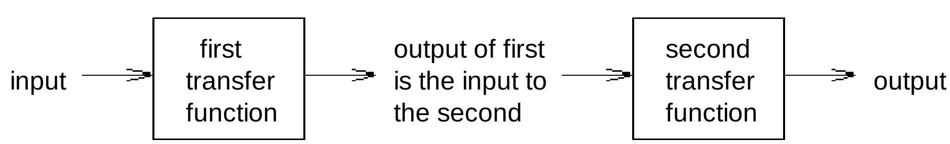


output = input x transfer function

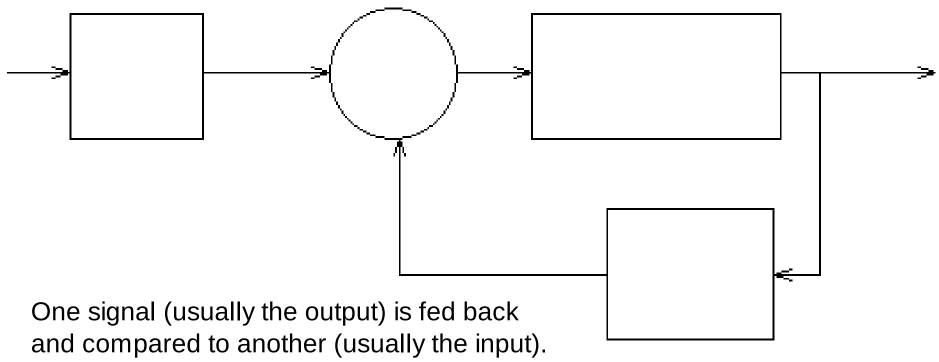
In this case, the transfer function = $\frac{\text{output}}{\text{input}}$

This is NOT always possible

Blocks can be "hooked" together, that is, the output of one block could be the input to another

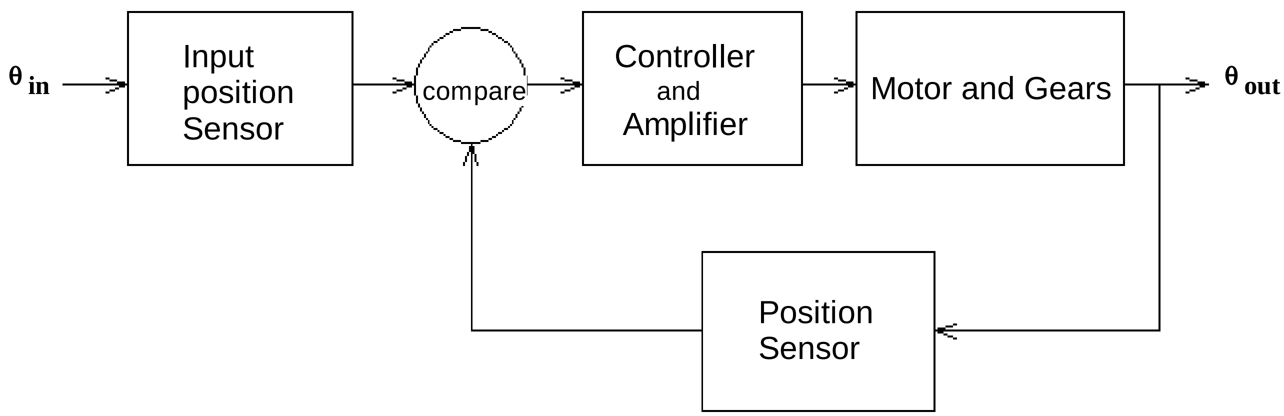


Blocks can be hooked together in more complex ways like this loop. This is an example "feedback".



One signal (usually the output) is fed back and compared to another (usually the input).

Example of a System (A Position Servo with Feedback)



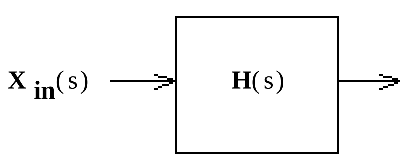
Again, the lines represent signals. Yes, there may also be considerable power moving from one block to another or out the end, but that's not what we'll care about here. All we really care about here is the basic information.

Blocks represent subsystems, devices or components which act upon an input or inputs to produce an output.

We will want a **mathmatical** way to represent the signals and the action of the blocks so that we can get a better handle on what's happening, and, hopefully, make the whole sytem work as we want.

We'll assume that each of the blocks is linear and time invariant. Anything else gets too hard too fast, and this is a good place to start. Many real devices can be modeled as linear and time invariant, at least over some region of operation.

For linear systems, where the signals and systems can be represented by Laplace transforms:



$$\mathbf{X}_{out}(s) = \mathbf{X}_{in}(s)\cdot\mathbf{H}(s)$$

Transfer function: $\mathbf{H}(s) = \dfrac{\mathbf{X}_{out}(s)}{\mathbf{X}_{in}(s)}$

$\mathbf{X}_{in}$ and $\mathbf{X}_{out}$ could be anything from small electrical signals to powerful mechanical motions or forces.

The variable "s" comes from Laplace transformations.

We will come back to this and spend a LOT more time on Laplace transforms and transfer functions

ECE 3510 Lecture 2 notes Introduction to Feedback Systems

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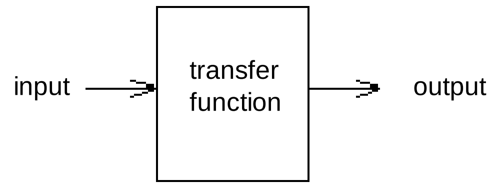
Yesterday we drew a block diagram on the board.
Let's examine those blocks a little more closely

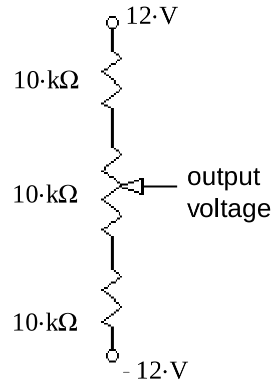


What's inside?

How are the input and output related? If you know the input, how do you find the output? Sometimes we can just multiply the input by the expression in the box to get the output. Then the expression in the box is called a transfer function.

In that case, the transfer function = $\dfrac{\text{output}}{\text{input}}$

A very simple case, a potentiometer



the input is θ , the angle of the shaft

We measure the voltage over its range of motion to find Kp.

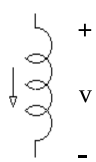
$$K_p = \frac{\text{output}}{\text{input}} = \frac{4\cdot V - -4\cdot V}{270\cdot deg} = 29.63\cdot\frac{mV}{deg} = 1.7\cdot\frac{V}{rad}$$

range of motion

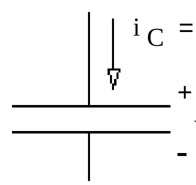
In this case, "zero" must be in the center of the range of motion

$$\theta_{in}(t) \longrightarrow \boxed{K_p = 1.7\cdot\frac{V}{rad} = 0.03\cdot\frac{V}{deg}} \longrightarrow V_{out}(t) = K_p\cdot\theta_{in}(t)$$

Nice... too bad it works for so few things in the time domain! Simple voltage dividers, amplifiers, and not much else. All real electrical systems also have inductors and capacitors.



$$\frac{1}{L}\cdot\int v_L\,dt = i_L \qquad v_L = L\cdot\frac{d}{dt}i_L$$



$$i_C = C\cdot\frac{d}{dt}v_C \qquad v_C = \frac{1}{C}\cdot\int i_C\,dt$$

We'll have to avoid capacitors and inductors-- they're too complicated... You can't just multiply when there are differentials involved

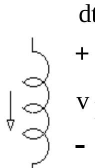
How about the mechanical world? $F = ma$, Great, no differentials... uh, except... $F = m\cdot a = m\cdot\frac{d}{dt}v = m\cdot\frac{d^2}{dt^2}x$

And then there are springs: $F = k\cdot x = k\cdot\int v\,dt = k\cdot\int\int a\,dt\,dt$

Isn't there some way that we could possibly replace all this differentiation and integration with multiplication and division?

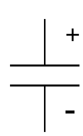
Laplace transforms $\frac{d}{dt}$ operation can be replaced with s, and $\int \blacksquare\,dt$ can be replaced by $\frac{1}{s}$

Then...



$$v_L = L\cdot\frac{d}{dt}i_L$$

$$\mathbf{V_L}(s) = L\cdot s\cdot\mathbf{I_L}(s)$$



$$v_C = \frac{1}{C}\cdot\int i_C\,dt$$

$$\mathbf{V_C}(s) = \frac{1}{C}\cdot\frac{1}{s}\cdot\mathbf{I_C}(s) \qquad \mathbf{Z_C} = \frac{1}{C\cdot s}$$

Inductive impedance: $\mathbf{Z_L} = L\cdot s$

Recall from your Ordinary Differential Equations class, the Laplace transform method of solving differential equations. The Laplace transform allowed you to change time-domain functions to frequency-domain functions.

1) Transform your signals into the frequency domain with the Laplace transform.

$$F(s) = \int_0^\infty f(t)\cdot e^{-s\cdot t}\,dt$$ Unilateral Laplace transform

2) Solve your differential equations with plain old algebra, where:

$\frac{d}{dt}$ operation can be replaced with s, and $\int \blacksquare\, dt$ can be replaced by $\frac{1}{s}$

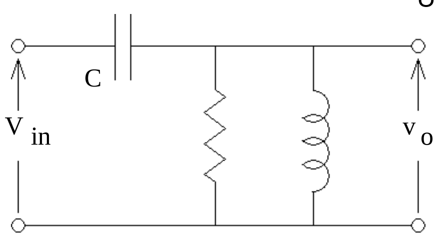
3) Transform your result back to the time domain with the inverse Laplace transform.

$$f(t) = \frac{1}{2\cdot\pi\cdot j}\cdot\int_{c-j\infty}^{c+j\infty} F(s)\cdot e^{s\cdot t}\,ds$$

OK, truth be told, we never actually use the inverse Laplace transform. We use tables instead.

Then our nice, linear, blocks could contain Laplace transfer functions, like this:

Consider a circuit:



Using the impedances in a voltage divider:

$$\mathbf{H}(s) = \frac{\mathbf{V_o}(s)}{\mathbf{V_{in}}(s)} = \frac{\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{L\cdot s}}}{\dfrac{1}{\dfrac{1}{R}+\dfrac{1}{L\cdot s}}+\dfrac{1}{C\cdot s}}\cdot\frac{\left(\dfrac{1}{R}+\dfrac{1}{L\cdot s}\right)}{\left(\dfrac{1}{R}+\dfrac{1}{L\cdot s}\right)} = \frac{1}{1+\dfrac{1}{C\cdot s\cdot R}+\dfrac{1}{C\cdot s\cdot L\cdot s}}\cdot\frac{\left(s^2\right)}{\left(s^2\right)}$$

$$= \frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{C\cdot L}}$$

This could now be represented as a block operator:

$$\mathbf{V_{in}}(s) \longrightarrow \boxed{\frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{C\cdot L}}} \longrightarrow \mathbf{V_o}(s) = \mathbf{V_{in}}(s)\cdot\mathbf{H}(s)$$

Transfer functions can be written for all kinds of devices and systems, not just electric circuits and the input and output do not have to be similar. For instance, the potentiometers used to measure angular position in the servo have an angle as input and a voltage as output.

Laplace transforms will be important!!

BUT, remember, the first step is to transform the signals into the frequency domain with the Laplace transform. Maybe we ought to deal with the signals first...

FIRST: Laplace transforms of <u>signals</u>

Let's evaluate some of these and see if we can make a table

Ex. 1 $f(t) = \delta(t)$ The Impulse or "Dirac" function, not a very likely signal in real life.

$$\mathbf{F}(s) = \int_0^\infty \delta(t)\cdot e^{-s\cdot t}\,dt$$

but: $\delta(t)\cdot g(t) = \delta(t)\cdot g(0)$ so:

any function

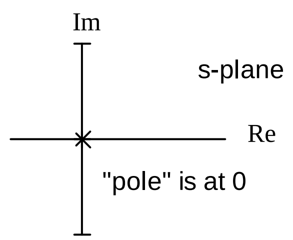
$$= \int_0^\infty \delta(t)\cdot e^{-s\cdot 0}\,dt = \int_0^\infty \delta(t)\cdot 1\,dt = 1$$

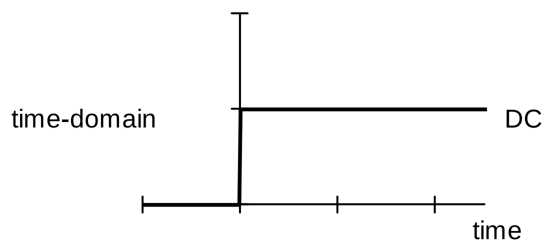
Ex. 2 $f(t) = u(t)$ The unit-step function, a constant value (DC) signal

$$\mathbf{F}(s) = \int_0^{\infty} u(t)\cdot e^{-s\cdot t}\,dt \qquad \text{but:} \quad u(t) = 1 \quad \text{from } 0 \text{ to } \infty$$

$$= \int_0^{\infty} 1\cdot e^{-s\cdot t}\,dt = \frac{1}{-s}\cdot e^{-s\cdot t}\Big|_0^{\infty} = \frac{1}{-s}\cdot \overbrace{e^{-s\cdot\infty}}^{0} - \frac{1}{-s}\cdot \overbrace{e^{-s\cdot 0}}^{1} = 0 - \frac{1}{-s}\cdot(1) = \frac{1}{s}$$

if $s > 0$





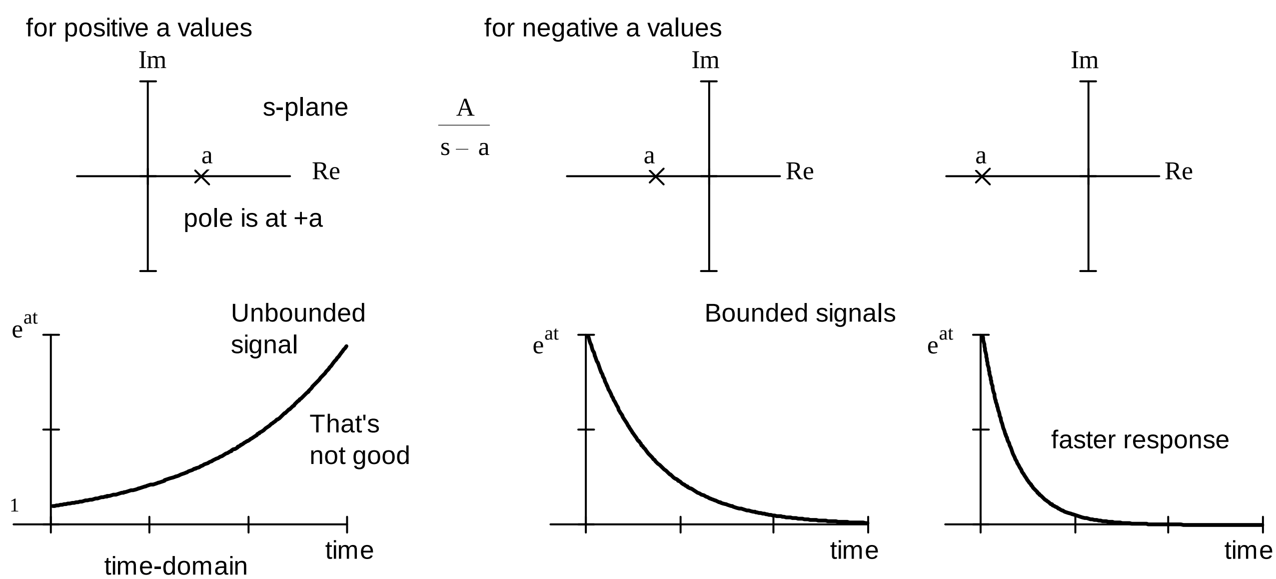
Ex. 3 $f(t) = u(t)\cdot e^{at}$

$$\mathbf{F}(s) = \int_0^{\infty} e^{at}\cdot e^{-s\cdot t}\,dt = \int_0^{\infty} e^{(a-s)\cdot t}\,dt = \frac{1}{(a-s)}\cdot e^{(a-s)\cdot t}\Big|_0^{\infty}$$

$$= \frac{1}{(a-s)}\cdot \overbrace{e^{(a-s)\cdot\infty}}^{0} - \frac{1}{(a-s)}\cdot \overbrace{e^{(a-s)\cdot 0}}^{1} = 0 - \frac{1}{(a-s)}\cdot(1) = \frac{1}{s-a}$$

"pole" is at +a

if $s > a$



This is the single most-important Laplace transform case. In fact we really don't need any others. Ex.1 can be thought of as this case with $a = -\infty$. Ex. 2 can be thought of as $a = 0$. And finally, all sinusoids can be made from exponentials if you let the poles (a) be complex. Remember Euler's equations...

Euler's equations $e^{j\cdot\omega\cdot t} = \cos(\omega t) + j\cdot\sin(\omega t)$ $\qquad e^{(\alpha\cdot t + j\cdot\omega\cdot t)} = e^{\alpha\cdot t}\cdot(\cos(\omega t) + j\cdot\sin(\omega t))$

Pole Location(s) correspond to the type of signal.

Euler's equations $\cos(\omega \cdot t) = \dfrac{e^{j\cdot\omega\cdot t} + e^{-j\cdot\omega\cdot t}}{2}$ $\sin(\omega \cdot t) = \dfrac{e^{j\cdot\omega\cdot t} - e^{-j\cdot\omega\cdot t}}{2\cdot j}$

Ex. 4 $f(t) = u(t)\cdot\cos(\omega\cdot t)$

$$\mathbf{F}(s) = \int_0^\infty \cos(\omega\cdot t)\cdot e^{-s\cdot t}\,dt = \int_0^\infty \left(\frac{e^{j\cdot\omega\cdot t} + e^{-j\cdot\omega\cdot t}}{2}\right)\cdot e^{-s\cdot t}\,dt = \int_0^\infty \frac{e^{(j\cdot\omega - s)\cdot t} + e^{-(j\cdot\omega + s)\cdot t}}{2}\,dt$$

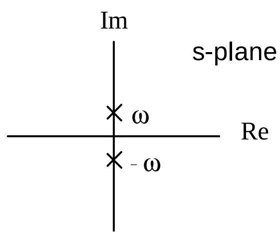
$$= \frac{1}{2}\cdot\int_0^\infty e^{(j\cdot\omega - s)\cdot t}\,dt + \frac{1}{2}\cdot\int_0^\infty e^{-(j\cdot\omega + s)\cdot t}\,dt$$

$$= \frac{1}{2}\cdot\left(\frac{1}{j\cdot\omega - s}\right)\cdot e^{(j\cdot\omega - s)\cdot t}\Bigg|_0^\infty + \frac{1}{2}\cdot\left[\frac{1}{-(j\cdot\omega + s)}\right]\cdot e^{-(j\cdot\omega + s)\cdot t}\Bigg|_0^\infty$$

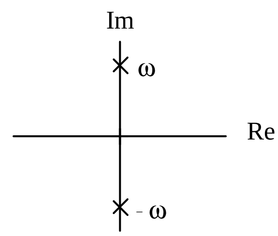
$$= 0 - \frac{1}{2}\cdot\left(\frac{1}{j\cdot\omega - s}\right)\cdot(1) + 0 - \frac{1}{2}\cdot\left[\frac{1}{-(j\cdot\omega + s)}\right]\cdot(1) = \frac{-1}{-2\cdot j\cdot\omega - 2\cdot s} + \frac{-1}{2\cdot j\cdot\omega - 2\cdot s}$$

$$= \frac{1}{2\cdot j\cdot\omega + 2\cdot s} + \frac{-1}{2\cdot j\omega - 2\cdot s} = \frac{(2\cdot j\cdot\omega - 2\cdot s) - (2\cdot j\cdot\omega + 2\cdot s)}{(2\cdot j\cdot\omega + 2\cdot s)\cdot(2\cdot j\cdot\omega - 2\cdot s)}$$

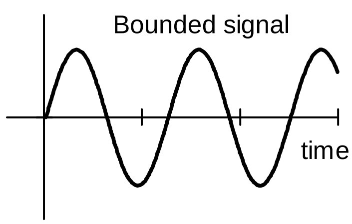
$$= \frac{-4\cdot s}{(2\cdot j\cdot\omega + 2\cdot s)\cdot(2\cdot j\cdot\omega - 2\cdot s)} = \frac{-4\cdot s}{4\cdot j^2\cdot\omega^2 - 4\cdot s^2} = \frac{-s}{-\omega^2 - s^2} = \frac{s}{\omega^2 + s^2}$$



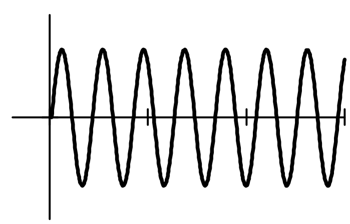
$$\frac{A\cdot s + B}{\left(s^2 + \omega^2\right)}$$



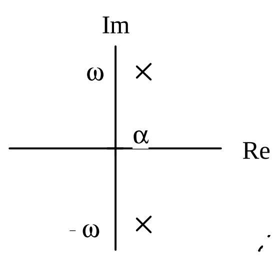
time-domain



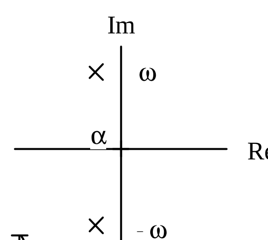
Doesn't converge



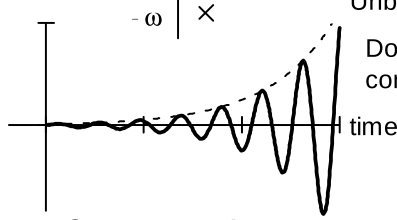
What if the poles have a real component? $f(t) = u(t)\cdot e^{\alpha\cdot t}\cdot\sin(\omega\cdot t)$



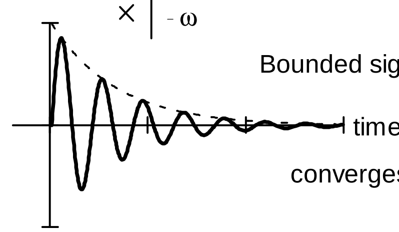
$$\frac{A\cdot s + B}{(s - \alpha)^2 + \omega^2}$$



Unbounded signal



Doesn't converge



Bounded signal

converges

Ex. 5 Multiply by time property

$$f(t) = u(t)\cdot t\cdot e^{a\cdot t} \qquad \mathbf{F}(s) = \int_0^\infty t\cdot e^{a\cdot t}\cdot e^{-s\cdot t}\,dt = \int_0^\infty t\cdot e^{(a-s)\cdot t}\,dt$$

Remember integration by parts:

$$\int h(t)\cdot\frac{d}{dt}g(t)\,dt = h(t)\cdot g(t) - \int g(t)\cdot\frac{d}{dt}h(t)\,dt$$

choose: $h(t) = t$ from which: $\frac{d}{dt}h(t) = 1$

and: $\frac{d}{dt}g(t) = e^{(a-s)\cdot t}$ from which: $g(t) = \int e^{(a-s)\cdot t}\,dt = \frac{e^{(a-s)\cdot t}}{(a-s)}$

$$h(t)\cdot g(t) \quad - \int g(t)\cdot\frac{d}{dt}h(t)\,dt$$

$$\mathbf{F}(s) = \int_0^\infty t\cdot e^{(a-s)\cdot t}\,dt = t\cdot\frac{e^{(a-s)\cdot t}}{(a-s)}\Bigg|_0^\infty - \int_0^\infty \frac{e^{(a-s)\cdot t}}{(a-s)}\cdot(1)\,dt = t\cdot\frac{e^{(a-s)\cdot t}}{(a-s)}\Bigg|_0^\infty - \frac{e^{(a-s)\cdot t}}{(a-s)^2}\Bigg|_0^\infty$$

$$= 0 - 0 - \left[0 - \frac{1}{(a-s)^2}\right]$$

$$= \frac{1}{(a-s)^2} = \frac{1}{(s-a)^2}$$

The easy way:

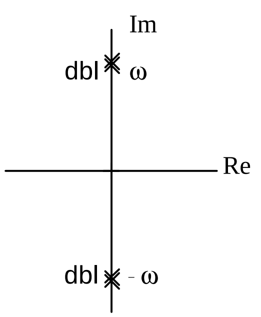
Use the "multiplication by time" property # 5 on p.8 of the Bodson textbook

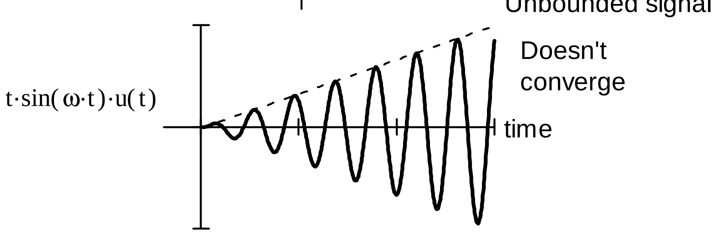
$$t\cdot x(t) \iff -\frac{d}{ds}X(s)$$

$$t\cdot e^{a\cdot t} \iff -\frac{d}{ds}\left(\frac{1}{s-a}\right) = -\frac{d}{ds}\left[(s-a)^{-1}\right] = -\frac{1}{-1}\cdot\frac{1}{(s-a)^2}\cdot\left[\frac{d}{ds}(s-a)\right] = \frac{1}{(s-a)^2}\cdot 1 = \frac{1}{(s-a)^2}$$

Anything that works for exponentials also works for sines and cosines...

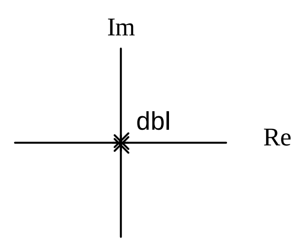
$$\frac{A\cdot s + B}{\left(s^2+\omega^2\right)^2}$$

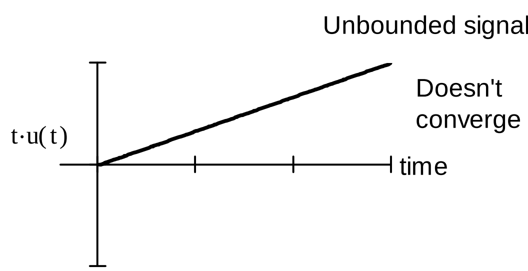




And "DC" too...

$$\frac{A}{s^2}$$

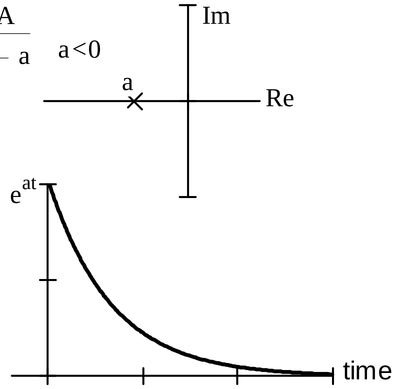


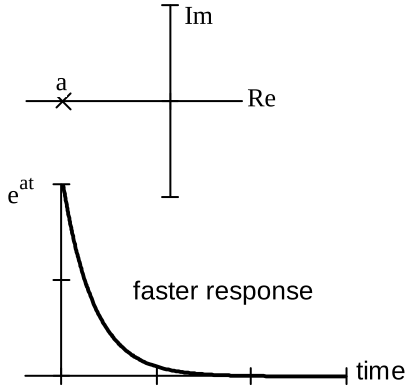


Signal Type, Boundedness, and Convergence can be predicted from the poles

Poles in the Open-Left-Half-Plane (OLHP) Real part of pole is negative $\mathrm{Re}(s_p) < 0$

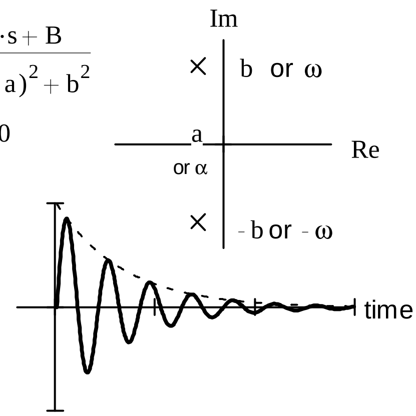
$\dfrac{A}{s-a}$ $a<0$





$\dfrac{A\cdot s+B}{(s-a)^2+b^2}$

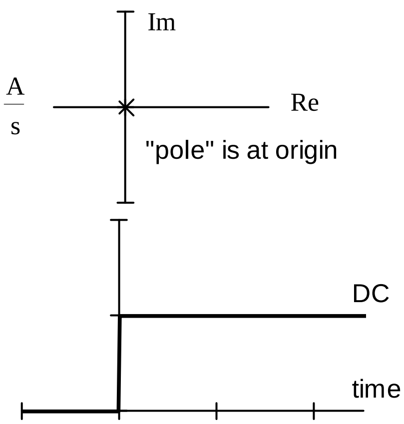
$a<0$



Bounded signals, Converge to zero

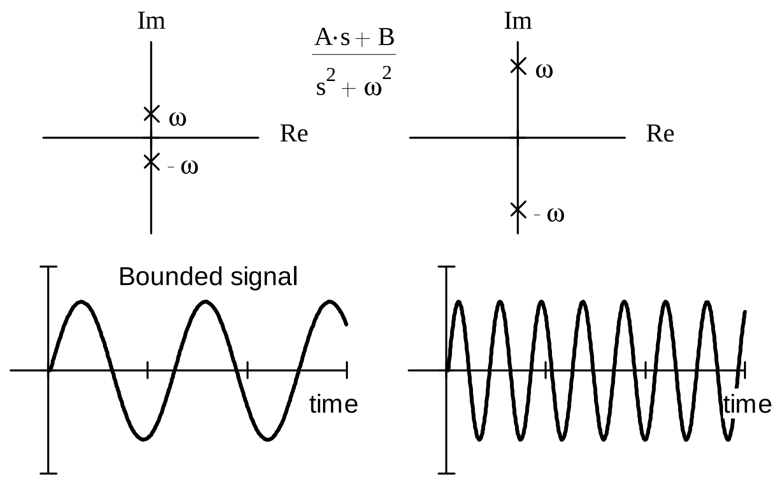
Single Poles on Imaginary Axis Real part of pole is zero $\mathrm{Re}(s_p) = 0$

$\dfrac{A}{s}$



Bounded signal,
Converges to DC value

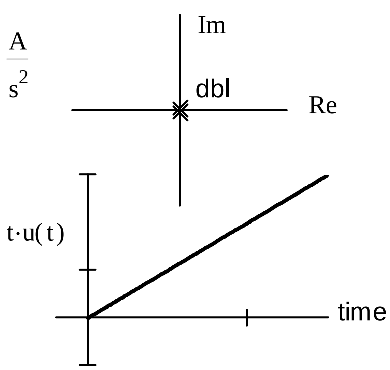
$\dfrac{A\cdot s+B}{s^2+\omega^2}$



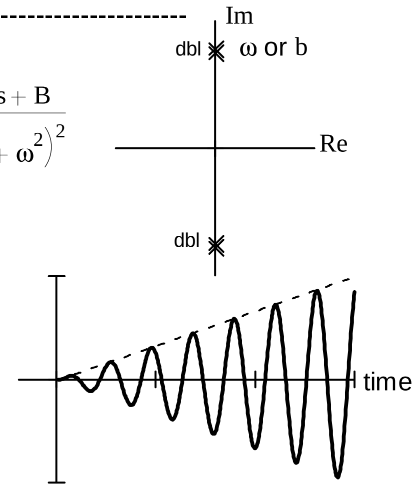
Bounded signals, Don't Converge

Double Poles on Imaginary Axis or

$\dfrac{A}{s^2}$



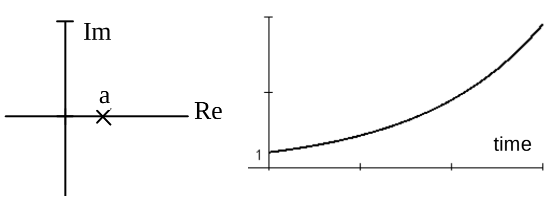
$\dfrac{A\cdot s+B}{\left(s^2+\omega^2\right)^2}$

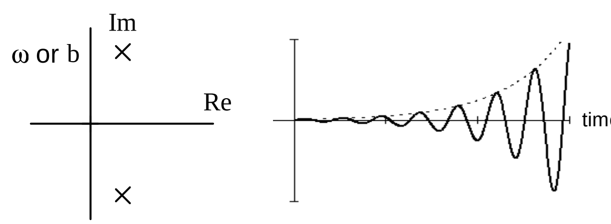


$t\cdot\sin(\omega\cdot t)\cdot u(t)$

and/or cosine

In the Open-Right-Half-Plane (ORHP)





Unbounded signals, Don't Converge

ECE 3510 Laplace Transforms (Unilateral)

$$f(t) = \frac{1}{2\cdot\pi\cdot j}\cdot\int_{c-j\infty}^{c+j\infty} F(s)\cdot e^{s\cdot t}\, ds \qquad\qquad F(s) = \int_{0}^{\infty} f(t)\cdot e^{-s\cdot t}\, dt$$

c is a constant within the region of convergence

	f(t)	F(s)	
1	$\delta(t)$	1	
2	$u(t)$	$\frac{1}{s}$	
3	$t\cdot u(t)$	$\frac{1}{s^2}$	
4	$t^n\cdot u(t)$	$\frac{n!}{s^{n+1}}$	
5a	$e^{a\cdot t}\cdot u(t)$	$\frac{1}{s-a}$	
5b	$e^{-\frac{t}{\tau}}\cdot u(t)$	$\frac{1}{s+\frac{1}{\tau}}$	$\tau = -\frac{1}{a}$ = time constant
6	$t\cdot e^{a\cdot t}\cdot u(t)$	$\frac{1}{(s-a)^2}$	
7	$t^n\cdot e^{a\cdot t}\cdot u(t)$	$\frac{n!}{(s-a)^{n+1}}$	
8a	$\cos(b\cdot t)\cdot u(t)$	$\frac{s}{s^2+b^2}$	$b = \omega$ = radian frequency
8b	$\sin(b\cdot t)\cdot u(t)$	$\frac{b}{s^2+b^2}$	
9a	$e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{s-a}{(s-(a+bj))\cdot(s-(a-bj))} = \frac{s-a}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$ $\frac{s-a}{(s-a)^2+b^2} = \frac{s-a}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$	
9b	$e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{b}{(s-(a+bj))\cdot(s-(a-bj))} = \frac{b}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$ $\frac{b}{(s-a)^2+b^2} = \frac{b}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$	
11a	$t\cdot e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{(s-a)^2-b^2}{\left[(s-a)^2+b^2\right]^2} = \frac{(s-a)^2-b^2}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$	
11b	$t\cdot e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{2\cdot b\cdot(s-a)}{\left[(s-a)^2+b^2\right]^2} = \frac{2\cdot b\cdot(s-a)}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$	

Euler's equations $\quad \cos(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}+e^{-j\cdot\omega\cdot t}}{2} \qquad \sin(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}-e^{-j\cdot\omega\cdot t}}{2\cdot j}$

ECE 3510 Laplace Properties

Operation	f(t)	F(s)
Addition	$f(t) + g(t)$	$F(s) + G(s)$
Scalar multiplication	$k \cdot f(t)$	$k \cdot F(s)$
Linearity	$k \cdot f(t) + n \cdot g(t)$	$k \cdot F(s) + n \cdot G(s)$
Time differentiation	$\frac{d}{dt} f(t)$	$s \cdot F(s) - f(0^-)$
	$\frac{d^2}{dt^2} f(t)$	$s^2 \cdot F(s) - s \cdot f(0^-) - \underbrace{\frac{d}{dt} f(0^-)}_{\text{initial slope}}$
	$\frac{d^3}{dt^3} f(t)$	$s^3 \cdot F(s) - s^2 \cdot f(0^-) - s \cdot \underbrace{\left(\frac{d}{dt} f(0^-)\right)}_{\text{initial slope}} - \underbrace{\frac{d^2}{dt^2} f(0^-)}_{\text{initial curvature}}$
Time integration	$\int_{0^-}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s)$
	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s) + \frac{1}{s} \cdot \int_{-\infty}^{0^-} f(t)\, dt$
Time shift	$f(t - t_0) \cdot u(t - t_0)$	$F(s) \cdot e^{-s \cdot t_0}$ $\quad t_0 \geq 0$
Frequency shift	$f(t) \cdot e^{s_0 \cdot t}$	$F(s - s_0)$
Frequency differentiation	$-t \cdot f(t)$	$\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s')\, ds'$
Scaling	$f(a \cdot t)$ $\quad a \geq 0$	$\frac{1}{a} \cdot F\left(\frac{s}{a}\right)$
Time convolution	$f(t) * g(t)$	$F(s) \cdot G(s)$
Frequency convolution	$f(t) \cdot g(t)$	$\frac{1}{2 \cdot \pi \cdot j} \cdot F(s) * G(s)$
Initial value	$f(0^+)$	$\lim_{s \to \infty} s \cdot F(s)$ $\quad n > m$ # of poles > # of zeroes
Final value	$f(\infty)$	$\lim_{s \to 0} s \cdot F(s)$ $\quad$ (all poles of $sF(s)$ in LHP)

ECE 3510 Laplace Properties