# ECE 3510

### Bode Plot Notes

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Steps to make Bode Plots

Sample transfer function: P(s) =

$$K \cdot \frac{\left(s+z_{1}\right) \cdot \left(s+z_{2}\right) \cdot \left(s+z_{3}\right)}{s^{2} \cdot \left(s+p_{1}\right) \cdot \left(s+p_{2}\right) \cdot \left(s^{2}+2 \cdot \zeta \cdot \omega_{n}+\omega_{n}^{2}\right)}$$

if complex pole is expressed:  $\left[\left(s+a\right)^2+b^2\right]$ 

then: 
$$\omega_n = \sqrt{a^2 + b^2}$$

1. a) Rewrite, replacing all s's with blanks:

$$P(\_) = K \cdot \frac{(\_+z_1) \cdot (\_+z_2) \cdot (\_+z_3)}{\_^2 \cdot (\_+p_1) \cdot (\_+p_2) \cdot (\_+\omega_n) \cdot (\_+\omega_n)}$$

notice that you also simplify the complex poles and/or zeros for now

2. a) Scale your frequency axis to start at some frequency less than your smallest pole or zero.

b) Plug this starting frequency in for any poles or zeros at the origin (write it in the blank as  $j\omega$ ).

$$P(j\omega, \_) = K \cdot \frac{(- + z_1) \cdot (- + z_2) \cdot (- + z_3)}{(j \cdot \omega_{start})^2 \cdot (- + p_1) \cdot (- + p_2) \cdot (- + \omega_n) \cdot (- + \omega_n)}$$

c) Ignore all the other blanks and calculate your initial magnitude, initial slope and the initial phase angle.

Initial magnitude = 
$$K \cdot \frac{(z_1) \cdot (z_2) \cdot (z_3)}{(j \cdot \omega_{start})^2 \cdot (p_1) \cdot (p_2) \cdot (\omega_n) \cdot (\omega_n)}$$

Initial slope

Initial phase

 $\omega$ 's in the numerator --> + 20dB/decade each  $\omega$ 's in the denominator --> - 20dB/decade each numerator,  $j => 90^{\circ}$ ,  $- => +180^{\circ}$ denominator,  $j => -90^{\circ}$ ,  $- => -180^{\circ}$ 

- 3. a) Extend the line to the first pole or zero.
  - b) Replace that pole or zero with  $j\omega$  and cross out the value of the pole or zero:

$$K \cdot \frac{\left(j\omega + z_{11}^{*}\right) \cdot \left(- + z_{21}\right) \cdot \left(- + z_{31}\right)}{\left(j\omega\right)^{2} \cdot \left(- + p_{11}\right) \cdot \left(- + p_{21}\right) \cdot \left(- + \omega_{n1}\right) \cdot \left(- + \omega_{n1}\right)}$$

OR 
$$K \cdot \frac{(-z_1) \cdot (-z_2) \cdot (-z_3)}{(j\omega)^2 \cdot (j\omega + \chi_1) \cdot (-p_2) \cdot (-\omega_n) \cdot (-\omega_n)}$$

c) Use this to find the new slope and phase angle.

Unless you replaced what was once a -s or crossed out a negative value:

zeros turn up the slope --> + 20dB/decade

poles turn down the slope --> - 20dB/decade

zeros increase the phase angle --> + 90 degpoles decrease the phase angle --> - 90deg

4. Repeat step 3 for each successive pole or zero.

After the last one you may want to check the magnitude or slope and phase again.

- 5. Draw a smooth line through the bode plots to estimate the actual magnitude and phase.
  - a) At poles and zeroes that are NOT complex

-3dB at single poles -6dB at double poles Magnitude effects extend Actual magnitude: etc.. +3dB at single zeros +6dB at double zeros about 1 decade fore and aft.

Angle effects extend about 1.5 decade fore and aft.









# ECE 3510 Bode Examples p.4 Ex. 7





# ECE 3510

**Ex.1** P(s) = ?





 $\mathbf{P}(s) = ?$ 



Bode Plot to Transfer Function Examples p.1

# Bode Plot to Transfer Function Examples p.2







 $\mathbf{P}(s) = ?$ 



### Bode Plot to Transfer Function Examples p.2



**Ex. 6** What if the phase plot was:

 $\mathbf{P}(s) = ?$ 





Note: A somewhat more involved method is outlined in Nise section 10.13 (p.660 in 3rd ed., 665 in 4th). That method involves estimating only one pole or zero at a time and then subtracting the effect from original to more clearly see the others. This can work much better with real experimental data. Real data always has delay effects and other non-linearities which make the process much harder.



#### ECE 3510 Gain, Phase, and Delay margins

ECE 3510





Gain Margin

Phase Margin

Delay Margin

Gain Margin

Phase Margin

Delay Margin

#### Using Frequency-domain (Bode Plot) Design for the Double Integrator Also the Basis of problem 3 in homework BP3

#### **Double Integrator**

A very common system and a difficult design problem.

It's Newton's fault: 
$$F = m \cdot a = m \cdot \frac{d^2}{dt^2} x$$
  
Same for angular motion:  $T = J \cdot \alpha = J \cdot \frac{d^2}{dt^2} \theta$   
 $x = \frac{1}{m} \cdot \left( \int \int F \, dt \, dt \right)$   
 $x = \frac{1}{m} \cdot \left( \int \int F \, dt \, dt \right)$   
 $X(s) = F(s) \cdot \frac{1}{m \cdot s^2}$   
 $R = \frac{1}{m \cdot s^2}$ 

This problem arises anytime force is the input and position is the output.

Force is the ONLY way to get the motion of any object to change, so yes, this is a common problem.

RL worse.

In the Inverted Pendulum lab, the movement of the base was simplified to a first-order system to avoid the difficulties that come from this very issue.

The example used in section 5.3.9 is a VERY REAL example.



Given the issues with a PD (the differentiator). lets use a Lead controller.

### ECE 3510 Bode Design p1

#### Lead controller

See section 5.3.9

$$\mathbf{C}(s) = \mathbf{k}_{\mathbf{C}} \cdot \frac{\mathbf{s} + \mathbf{b}}{\mathbf{s} + \mathbf{a}}$$



Put the two together,

$$\mathbf{G}(s) = k_{c} \cdot \frac{s+b}{s+a} \cdot \frac{k_{p}}{s^{2}} = k_{p} \cdot k_{c} \cdot \frac{s+b}{s^{2} \cdot (s+a)}$$

But now the maximum phase angle difference from 180 doesn't occur where the magnitude crosses 0dB.

This problem is resolved in the math shown in the book, which makes:

$$\omega_c = \omega_p$$



### The Bottom Line

I've combined information from the table on page 155 with table on page 152.

	For double integrator problem				
	(a)	/	$_{/}$ approximation from simpler system of section 5.3.7		
	$\left(\frac{\mathbf{a}}{\mathbf{b}}\right)$	$\phi_p = PM$	ζ	%OS = PO = percent overshoot based on $\zeta$ approx.	
. Select your a/b ratio,	5.83	45 <sup>°</sup>	0.44	20.5.%	
single number in	9	53.1°	0.55	14.%	
following equations.	13.9	$60^{\circ}$	0.6	9.5.%	
use $\left(rac{a}{b} ight)$ as a single number	Or use eq. 5.67		Extension of table using approximate relationship between PM and overshoot developed in section 5.3.7		

2. Use eq. 5.69 to relate  $\omega_c$  to  $k_p$  and  $k_c$ .  $\frac{k_p \cdot k_c}{\omega_c^2} \cdot \sqrt{\frac{b}{a}} = 1$ 

between PM and overshoot developed in section 5.3.

OR, rearranged: 
$$\omega_p = \omega_c = \sqrt{k_p \cdot k_c} \cdot \sqrt{\frac{b}{a}}$$
  
nts may be useful: Note:  $\frac{b}{a} =$ 

Depending on your knowns and unknowns, other rearrangements may be useful:

$$k_{p} \cdot k_{c} = \omega_{c}^{2} \cdot \sqrt{\frac{a}{b}}$$
  $k_{p} = \frac{\omega_{c}^{2}}{k_{c}} \cdot \sqrt{\frac{a}{b}}$   $k_{c} = \frac{\omega_{c}^{2}}{k_{p}} \cdot \sqrt{\frac{a}{b}}$ 

To get answers and plots for BD5, prob.3, I arbitrarily used:

$$k_p := 1$$
 and found  $k_c$  from the eq

<u>a</u>

3. Find: 
$$a = \omega_c \cdot \sqrt{\frac{a}{b}} = \omega_p \cdot \sqrt{\frac{a}{b}}$$

the pole location of C(s)

$$b = \omega_c \cdot \sqrt{\frac{b}{a}} = \omega_p \cdot \sqrt{\frac{b}{a}}$$

 $\omega_c = 10$ 

the zero location of C(s)

# Why Bode Plots?

- 1. Provides a method to find the approximate transfer function as used in the Flexible Beam lab.
- 2. Terms GM and PM are in wide use and you need to know what they mean.
- 3. Sometimes used for design method as in the Flexible Beam lab.

You will find from BD5, prob.3, that the approximations of overshoot given in the table above are not very good (off by about a factor of 2), but, those predicted by the second-order approximation are even worse (b/c of zero close to origin).

### Name: \_\_\_\_\_

A.Stolp

d

1. Sketch the Bode plots for the following transfer functions. Label the graphs, give the slopes of the lines in the magnitude plot and draw the "smooth" lines.

a) 
$$P(s) = \frac{s+10}{(s+1)\cdot(s+100)}$$





$$s \cdot (s + 400)$$

c) P(s) =  $\frac{800 \cdot (s - 10) \cdot (s + 10)}{(s + 0.1) \cdot (s + 400)^2}$ 

plot on next page



d) P(s) = 
$$\frac{900}{(s+30)^2 \cdot (s+1)}$$

1. Sketch the Bode plots for the following transfer functions. Label the graphs, give the slopes of the lines in the magnitude plot and draw the "smooth" lines. s + 50

a) 
$$P(s) = \frac{s+50}{s^2+0.4\cdot s+4}$$



b) 
$$P(s) = \frac{s^2 + 2 \cdot s + 100}{s^2}$$
 plot on next page



**Answers** 













1•10<sup>4</sup>

ECE 3510 Homework BP1 p7

#### Name: \_\_

Homework # BP2 Due: Thur., 4/6 c

1

10

 $\omega$  (rad/sec)

100

You must show the work needed to get the answers below. Add your own paper if necessary.

- 1. (a & c are from Problem 5.2 in Bodson text.)
- a) The magnitude Bode plot of a system is shown below. What are the possible transfer functions of stable systems having this Bode plot?



5

0<u>-</u> 0.1





# ECE 3510 Homework Bd2 p.3

2. The system whose Bode plots are given at right is stable in closed-loop. Find its gain margin, phase margin, and delay margin. Show your work on the drawings.



- 3. Problem 5.3 in the text.
  - a) The system whose Bode plots are given at left is stable in closed-loop.
     Find its gain, phase, and delay margins. Show your work on the drawings.



b) Describe the behavior of the closed-loop system of part (a) if the open-loop gain is increased to a value close to the maximum value given by the gain margin. In particular, what can you say about the locations of the poles of the closed-loop system?

c) Consider an open-loop stable system which is such that the magnitude of its frequency response, including the gain factor k, is less than 1 for all  $\omega$  ( |kG(s)| < 1). Can you determine whether the closed-loop system is stable with only that information? If yes, show how.

4. 4. Problem 5.13 b & new c & d (p.149) in the text.

### ECE 3510 Homework BP2 p.5

b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the left. For this system:

• How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?

• What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.

• How much time delay can there be in feedback system before the phase margin disappears.



c ) For the system of part (a), give the steady-state response of the open-loop system an input  $x(t) = 4\cos(10t)$ . express the answer in the time-domain.

d) Give the steady-state response of the closed-loop system for the same input. Hint: closed loop output is:  $\begin{array}{c} \text{input} \cdot \frac{G(10 \cdot j)}{1 + G(10 \cdot j)} \end{array}$ 

# ECE 3510 Homework BP2 p.6

- 5. Like problem 5.9a (p.147) in the Bodson text.
  - a) Give the gain margin, the phase margin and the delay margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).



AnswersThe rest are NOT stable1.a) 
$$P(s) = 10 \cdot \frac{s+1}{s+10}$$
 $10 \cdot \frac{s-1}{s+10}$  $-10 \cdot \frac{s-1}{s+10}$  $10 \cdot \frac{s-1}{s+10}$  $10 \cdot \frac{s-1}{s-10}$  $10 \cdot \frac{s-1}{s-10}$  $10 \cdot \frac{s-1}{s-10}$  $-10 \cdot \frac{s-1}{s-10}$ b)  $P(s) = \frac{10 \cdot s \cdot (s+2)}{(s^2+10 \cdot s+10000)}$ c)  $P(s) = \frac{10 \cdot (s+0.1) \cdot (s^2+0.4 \cdot s+4)}{s \cdot (s^2+0.2 \cdot s+1) \cdot (s^2+2 \cdot s+400)}$ 2.  $GM \simeq 25 \cdot dB$ PM  $\simeq 120 \cdot deg$ DM := 600 \cdot ms3. a)  $GM \simeq 21 \cdot dB$ PM  $\simeq 50 \cdot deg$ DM := 2.6 \cdot secb) The system will have a transient ring at about 10 rad/sec.  
Two poles of the closed loop system will be close to  $\pm 10j$ .c) Yes, it must be stable. Prove by closed-loop  
transfer function, Bode gain margin or Nyquist:  
 $N=0, P=0, Z=0$ 4. b) Gain may be increased by  $\simeq 2dB$  and reduced by  $\simeq 4.4dB$ .  
 $c)  $2 \cdot \cos(10 \cdot t+158 \cdot deg)$ PM  $\simeq 40 \cdot deg$ DM  $\simeq 50 \cdot ms$ ECE 35105. a)  $GM \simeq 30 \cdot dB$ PM  $\simeq 40 \cdot deg$ DM  $\simeq 50 \cdot ms$ ECE 3510Homework BP2p.6$ 

#### Name:

1. Problem 5.13 b & new c & d in the text.

b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the left. For this system:

• How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?

• What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.

• How much time delay can there be in feedback system before the phase margin disappears.



c ) For the system of part (a), give the steady-state response of the open-loop system an input  $x(t) = 4\cos(10t)$ . express the answer in the time-domain.

d) Give the steady-state response of the closed-loop system for the same input. Hint: closed loop output is:  $\begin{array}{c} \text{input} \cdot \frac{G(10 \cdot j)}{1 + G(10 \cdot j)} \\ \end{array}$ 

### ECE 3510 Homework BP3 p.2

- 2. Like problem 5.9a in the Bodson text.
  - a) Give the gain margin, the phase margin and the delay margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).



Add another sheet of paper the following:

3. A system has a delay	of $D = 0.01 \cdot sec$	How many degrees of	phase does this represent at:
<b>a)</b> f := 1 ⋅ Hz	$f = 10 \cdot Hz$	$f = 100 \cdot Hz$	$\mathbf{f} := 1 \cdot \mathbf{k} \mathbf{H} \mathbf{z}$
b) $\omega := 1 \cdot \frac{\text{rad}}{1 - 1}$	$\omega := 10 \cdot \frac{\text{rad}}{10}$	$\omega := 100 \cdot \frac{\text{rad}}{\text{cm}}$	$\omega = 1000 \cdot \frac{\text{rad}}{\text{cm}}$
sec	sec	sec	sec

4. a) If the phase response of a pure time delay were plotted on linear phase vs. linear frequency plot, what would be the shape of the curve?

b) If the phase response of a pure time delay were plotted on linear phase vs. logarithmic frequency plot, what would be the shape of the curve?

Find your textbook from your electronics class (ECE 2280 here at the U). ECE 3510 Homework BP3 p.3 Find the chapter or section which covers feedback in amplifiers. Read the sections covering bandwidth or frequency response, poise reduction, distortion, reduction, and gain reduction.

sections covering bandwidth or frequency response, noise reduction, distortion reduction and gain reduction.

#### Amplifier Compensation

- 5. The plot at right shows the frequency response of an LF353 op amp.
  - a) Find the gain-bandwidth product (GBW).
  - b) Find  $A_0$  in both dB and as a factor.
  - c) Find the open-loop roll-off point and the compensation pole location.



- d) A voltage (series-shunt) feedback network is used to feed back 0.1% of the output back to the input in a negative manner. Find the closed-loop gain (as a factor and in dB) and the closed-loop roll-off point. Draw the closed-loop frequency response on the drawing above.
- e) Now use two equal amplifier stages (two op amps) to achieve the same gain as part in d), Find the closed-loop roll-off point of a single stage. Draw the closed-loop frequency response of a single stage on the drawing above.

Would this also be the 3dB roll-off point of the entire two-stage amplifier? If not, why not?

#### <u>Answers</u>

- 1. b) Gain may be increased by  $\geq 2dB$  and reduced by  $\geq 4.4dB$ .c)  $2 \cdot \cos(10 \cdot t + 158 \cdot deg)$ d)  $3.5 \cdot \cos(10 \cdot t + 140 \cdot deg)$
- 2. a)  $GM \simeq 30 \cdot dB$   $PM \simeq 40 \cdot deg$   $DM \simeq 50 \cdot ms$
- 3. a) 3.6·deg
   36·deg
   b) 0.573·deg
   5.73·deg

   360·deg
   3600·deg
   57.3·deg
   573·deg
- 4. a) A straight line of negative slope,  $\omega D$ , where D is the time delay.
  - b) A negative sloping line with a slope of  $\omega D$ . Since the frequency increases by a factor of 10 each decade, so would the downward slope of the line.
- 5. a)  $4 \cdot \text{MHz}$  b)  $110 \cdot \text{dB}$   $3.162 \cdot 10^5$  c)  $12.65 \cdot \text{Hz}$  pole:  $\frac{1}{s + 79.5}$ d)  $1000 \quad 60 \cdot \text{dB}$   $4 \cdot \text{kHz}$  e)  $126.5 \cdot \text{kHz}$  The 2-tB rell off point.

ECE 3510 Homework BP3 p.3

The 3dB roll-off point of the entire two-stage amplifier is a bit less than 126.5kHz because that would actually be a 6dB roll-off point.

