Using Frequency-domain (Bode Plot) Design for the Double Integrator

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4/11/23

Double Integrator

A very common system and a difficult design problem.

It's Newton's fault: $F = m\cdot a = m\cdot\frac{d^2}{dt^2}x$ $\qquad x = \frac{1}{m}\cdot\left(\int\int F\,dt\,dt\right)$ $\qquad \mathbf{X}(s) = \mathbf{F}(s)\cdot\frac{1}{m\cdot s^2}$

Same for angular motion: $T = J\cdot\alpha = J\cdot\frac{d^2}{dt^2}\theta$ $\qquad$ & $\mathbf{P}(s) = \frac{1}{m\cdot s^2}$

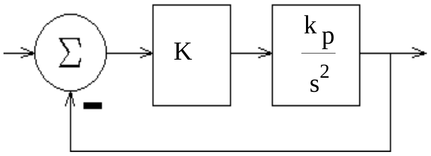
This problem arises anytime force is the input and position is the output.

Force is the ONLY way to get the motion of any object to change, so yes, this is a common problem.

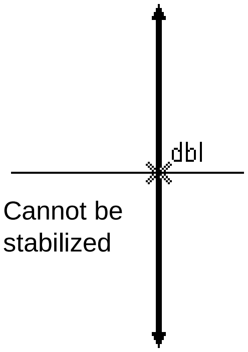
In the Inverted Pendulum lab, the movement of the base is simplified to a first-order system to avoid the difficulties that come from this very issue.

The example used in section 5.3.9 is a VERY REAL example.

in general: $\mathbf{P}(s) = \frac{k_p}{s^2}$



Root-locus for the double-integrator in a closed-loop (CL) feedback system.

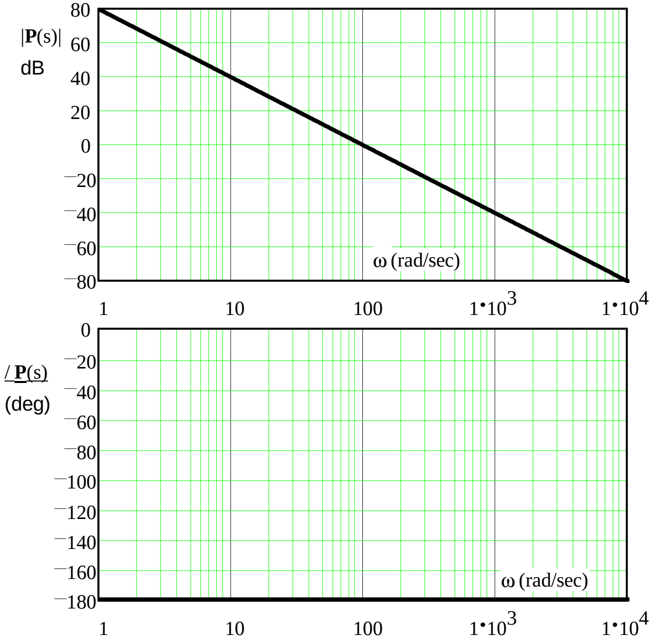


Cannot be stabilized

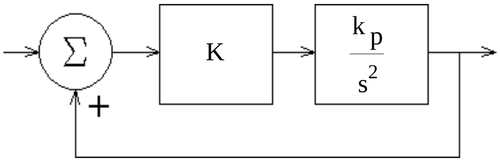
$$\mathbf{H}(s) = \frac{K\cdot\frac{k_p}{s^2}}{1 + K\cdot\frac{k_p}{s^2}}\cdot\left(\frac{s^2}{s^2}\right) = \frac{K\cdot k_p}{s^2 + K\cdot k_p}$$

MUST use a compensator.

Bode plots (numbers are representative)

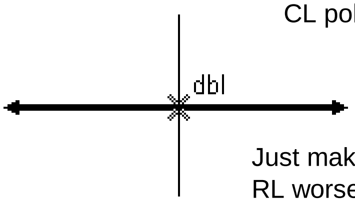


If the angle is always 180, then wouldn't positive feedback work?



$$\mathbf{H}(s) = \frac{K\cdot\frac{k_p}{s^2}}{1 - K\cdot\frac{k_p}{s^2}}\cdot\left(\frac{s^2}{s^2}\right) = \frac{K\cdot k_p}{s^2 - K\cdot k_p}$$

CL poles ± $\sqrt{K\cdot k_p}$



Positive feedback is similar to negative gain, which makes root-locus rules work backwards, here the real-axis rule:

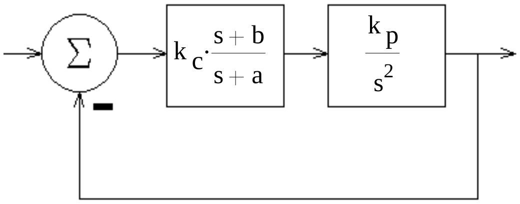
Just makes the RL worse.

Given the issues with a PD (the differentiator). lets use a Lead controller.

Lead controller

See section 5.3.9

$$\mathbf{C}(s) = k_c \cdot \frac{s+b}{s+a}$$



Put the two together,

$$\mathbf{G}(s) = k_c \cdot \frac{s+b}{s+a} \cdot \frac{k_p}{s^2} = k_p \cdot k_c \cdot \frac{s+b}{s^2 \cdot (s+a)}$$

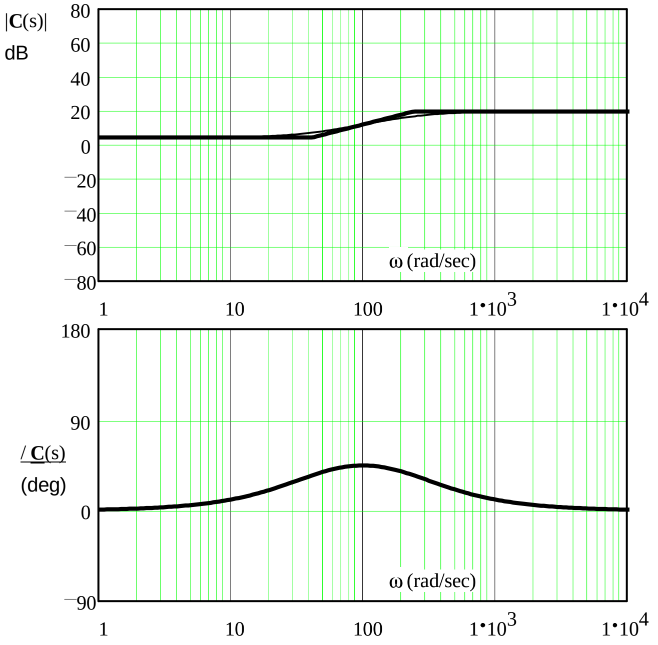
But now the maximum phase angle difference from 180 doesn't occur where the magnitude crosses 0dB.

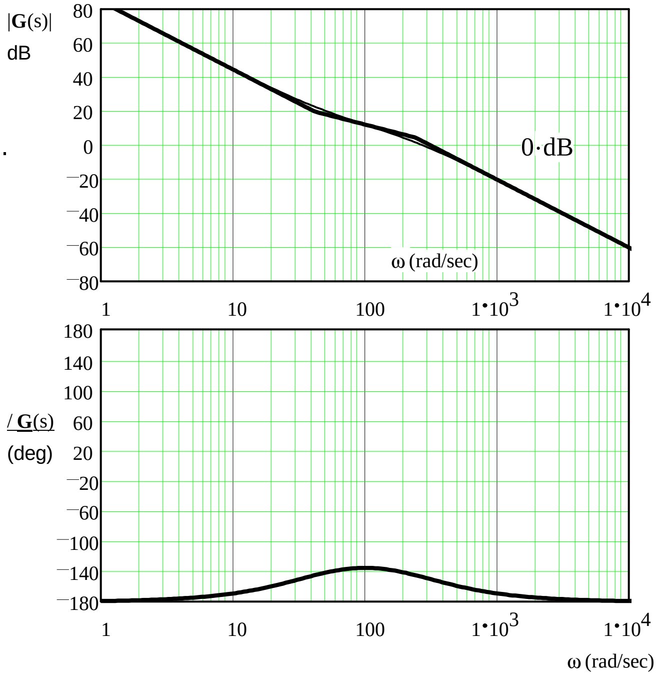
This problem is resolved in the math shown in the book, which makes:

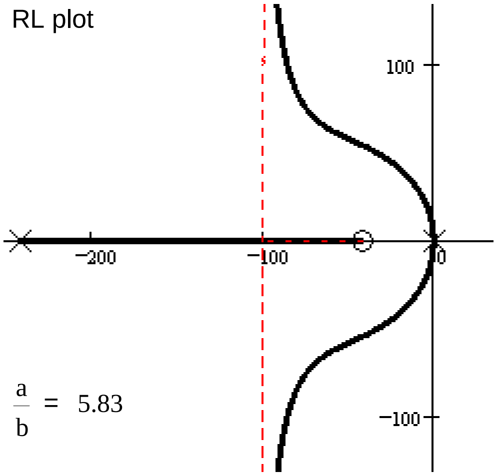
$$\omega_c = \omega_p$$

freq. of maximum phase difference = freq. where **G**(s) crosses 0dB.

Bode plots (numbers are representative)







$$\frac{a}{b} = 5.83$$

The Bottom Line

I've combined information from the table in section 5.3.7 with the table in section 5.3.9.

For double integrator problem

approximation from simpler system of section 5.3.7

1. Select your a/b ratio, use this ratio as a single number in following equations.

$\left(\frac{a}{b}\right)$	$\phi_p = \text{PM}$	ζ	$\%\text{OS} = \text{PO}$
5.83	45^o	0.44	$20.5\cdot\%$
9	53.1^o	0.55	$14\cdot\%$
13.9	60^o	0.6	$9.5\cdot\%$

= percent overshoot based on ζ approx.

PM, ζ relationship is also shown in section 5.3.7, 2nd eq. (5.63)

use $\left(\frac{a}{b}\right)$ as a single number

Or use eq. 5.73

Extension of table using approximate relationship between PM and overshoot developed in section 5.3.7

2. Use eq. 5.75 to relate ω_c to k_p and k_c. $\frac{k_p \cdot k_c}{\omega_c^2} \cdot \sqrt{\frac{b}{a}} = 1$ OR, rearranged: $\omega_p = \omega_c = \sqrt{k_p \cdot k_c \cdot \sqrt{\frac{b}{a}}}$

Note: $\frac{b}{a} = \frac{1}{\left(\frac{a}{b}\right)}$

Depending on your knowns and unknowns, other rearrangements may be useful:

$$k_p \cdot k_c = \omega_c^2 \cdot \sqrt{\frac{a}{b}} \qquad k_p = \frac{\omega_c^2}{k_c} \cdot \sqrt{\frac{a}{b}} \qquad k_c = \frac{\omega_c^2}{k_p} \cdot \sqrt{\frac{a}{b}}$$

To get some answers, I arbitrarily used: $\omega_c := 10$ $\quad k_p := 1$ and found k_c from the eq. above

3. Find: $a = \omega_c \cdot \sqrt{\frac{a}{b}} = \omega_p \cdot \sqrt{\frac{a}{b}}$ $\qquad b = \omega_c \cdot \sqrt{\frac{b}{a}} = \omega_p \cdot \sqrt{\frac{b}{a}}$

the pole location of $\mathbf{C}(s)$ $\qquad$ the zero location of $\mathbf{C}(s)$

Problem 5.14 in the text shows that the approximations of overshoot given in the table above are not very good (off by about a factor of 2), but, those predicted by the second-order approximation are even worse (b/c of zero close to origin).

Why Bode Plots?

1. Provides a method to find the approximate transfer function as used in the Flexible Beam lab.
2. Terms GM and PM are in wide use and you need to know what they mean.
3. Sometimes used for design method as in the Flexible Beam lab.

Example Problem 5.14 in the text.

a) Consider the lead controller for the double integrator. For the design that makes the crossover frequency equal to ω_C, obtain the polynomial that specifies the closed-loop poles (as a function of a/b and ω_C). Show that one closed-loop pole is at $s = -\omega_C$ no matter what a/b is.

$$\mathbf{G_c}(s) = \mathbf{P}(s)\cdot\mathbf{C}(s) = \frac{k_p}{s^2}\cdot k_c\cdot\frac{s+b}{s+a}$$

Denominator of the closed-loop transfer function: $\mathbf{D_G} + \mathbf{N_G} = s^2\cdot(s+a) + k_p\cdot k_c\cdot(s+b)$

$$= s^3 + a\cdot s^2 + k_p\cdot k_c\cdot(s+b) \quad = 0$$

to find poles

Substitute: $a = \omega_c\cdot\sqrt{\frac{a}{b}}$ $\quad b = \omega_c\cdot\sqrt{\frac{b}{a}}$ $\quad k_c = \frac{\omega_c^2}{k_p}\cdot\sqrt{\frac{a}{b}}$ eq. 5.70 in book.

$$0 = s^3 + \omega_c\cdot\sqrt{\frac{a}{b}}\cdot s^2 + k_p\cdot\left(\frac{\omega_c^2}{k_p}\cdot\sqrt{\frac{a}{b}}\right)\cdot\left(s + \omega_c\cdot\sqrt{\frac{b}{a}}\right) = s^3 + \omega_c\cdot\sqrt{\frac{a}{b}}\cdot s^2 + \omega_c^2\cdot\sqrt{\frac{a}{b}}\cdot s + \omega_c^2\cdot\sqrt{\frac{a}{b}}\cdot\omega_c\cdot\sqrt{\frac{b}{a}}$$

$$= s^3 + \omega_c\cdot\sqrt{\frac{a}{b}}\cdot s^2 + \omega_c^2\cdot\sqrt{\frac{a}{b}}\cdot s + \omega_c^3$$

Polynomial division:

$$s^2 + \omega_c\cdot\left(\sqrt{\frac{a}{b}} - 1\right)\cdot s + \omega_c^2$$

$$s + \omega_c \;\Big)\; s^3 + \omega_c\cdot\sqrt{\frac{a}{b}}\cdot s^2 + \omega_c^2\cdot\sqrt{\frac{a}{b}}\cdot s + \omega_c^3$$

$$-s^3 - \omega_c\cdot s^2$$

$$\left(\omega_c\cdot\sqrt{\frac{a}{b}} - \omega_c\right)\cdot s^2 + \omega_c^2\cdot\sqrt{\frac{a}{b}}\cdot s + \omega_c^3$$

$$-\omega_c\cdot\left(\sqrt{\frac{a}{b}} - 1\right)\cdot s^2 - \omega_c^2\cdot\left(\sqrt{\frac{a}{b}} - 1\right)\cdot s$$

$$\omega_c^2\cdot s + \omega_c^3$$

$$\omega_c^2\cdot s + \omega_c^3$$

$$0$$

No remainder, QED

b) Compute the other closed-loop poles, as functions of ω_C, when $a/b = 5.83$, 9, and 13.9.

The "other" roots are the roots of the quotient. $0 = s^2 + \omega_c\cdot\left(\sqrt{\frac{a}{b}} - 1\right)\cdot s + \omega_c^2$

$a/b = 5.83$ $\quad \left(\sqrt{\frac{a}{b}} - 1\right) = \left(\sqrt{5.83} - 1\right) = 1.415$

$$s = \left[\frac{-\omega_c\cdot\left(\sqrt{5.83} - 1\right)}{2} + \frac{1}{2}\cdot\sqrt{\left[\omega_c\cdot\left(\sqrt{5.83} - 1\right)\right]^2 - 4\cdot\omega_c^2}\right] = \frac{-\omega_c\cdot\left(\sqrt{5.83} - 1\right)}{2} + \frac{1}{2}\cdot\omega_c\cdot\sqrt{\left(\sqrt{5.83} - 1\right)^2 - 4}$$

$$= \frac{-\left(\sqrt{5.83} - 1\right)}{2} + \frac{1}{2}\cdot\sqrt{\left(\sqrt{5.83} - 1\right)^2 - 4} = -0.707 + 0.707j$$

$(-0.7071 - 0.7071\cdot j)\cdot\omega_c$ & $(-0.7071 + 0.7071\cdot j)\cdot\omega_c$

$a/b = 9$ $\quad \frac{-\left(\sqrt{9} - 1\right)}{2} = -1$ $\quad \frac{\sqrt{\left(\sqrt{9} - 1\right)^2 - 4}}{2} = 0$ $\quad -\omega_c$ & $-\omega_c$

$a/b = 13.9$ $\quad \frac{-\left(\sqrt{13.9} - 1\right)}{2} + \frac{1}{2}\cdot\sqrt{\left(\sqrt{13.9} - 1\right)^2 - 4} = -0.436$ $\quad \frac{-\left(\sqrt{13.9} - 1\right)}{2} - \frac{1}{2}\cdot\sqrt{\left(\sqrt{13.9} - 1\right)^2 - 4} = -2.292$

$-0.436\cdot\omega_c$ & $-2.292\cdot\omega_c$

For plots: $t := 0.01, 0.02 .. 1.5$

c) Use Matlab SISO or other software of your choice to confirm the results of part c) and the % overshoot figures expected from the phase margins by the second-order approximation. $(\ 20\cdot\% \quad 14\cdot\% \quad 9.5\cdot\% \ )$

$$\mathbf{P}(s)\cdot\mathbf{C}(s) = \frac{k_p}{s^2}\cdot k_c\cdot\frac{s+b}{s+a} \qquad H(s) = \frac{P(s)\cdot C(s)}{1+P(s)\cdot C(s)} = \frac{k_p\cdot k_c\cdot(s+b)}{(s+a)\cdot s^2 + k_p\cdot k_c\cdot(s+b)}$$

$\mathbf{X}(s) = \frac{1}{s}$ the unit step function

$$\mathbf{Y}(s) = \mathbf{X}(s)\cdot\mathbf{H}(s) = \frac{1}{s}\cdot\frac{k_p\cdot k_c\cdot(s+b)}{(s+a)\cdot s^2 + k_p\cdot k_c\cdot(s+b)} = \frac{1}{s}\cdot\frac{k_p\cdot k_c\cdot(s+b)}{(s+a)\cdot s^2 + k_p\cdot k_c\cdot(s+b)}$$

$$= \frac{1}{s}\cdot\frac{k_p\cdot k_c\cdot(s+b)}{\left(s+\omega_c\right)\cdot\left[s^2+\omega_c\cdot\left(\sqrt{\frac{a}{b}}-1\right)\cdot s+\omega_c^{\ 2}\right]}$$

I will set: $\omega_c := 10 \qquad k_p := 1$

First case: $\frac{a}{b} = a\_b := 5.83 \qquad a = \omega_c\cdot\sqrt{\frac{a}{b}} = a := \omega_c\cdot\sqrt{a\_b} \qquad a = 24.145$

$$b = \frac{\omega_c}{\sqrt{\frac{a}{b}}} = b := \frac{\omega_c}{\sqrt{a\_b}} \qquad b = 4.142$$

$$k_c = \frac{\omega_c^{\ 2}}{k_p}\cdot\sqrt{\frac{a}{b}} = k_c := \frac{\omega_c^{\ 2}}{k_p}\cdot\sqrt{a\_b} \qquad k_c = 241.454$$

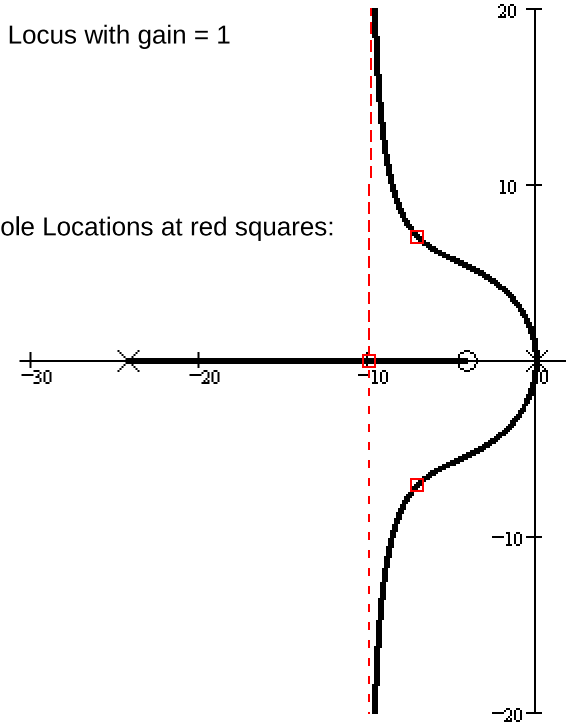
$$\mathbf{G}_c(s) = \frac{k_p}{s^2}\cdot k_c\cdot\frac{s+b}{s+a} = \frac{241.454\cdot(s+4.142)}{s^2\cdot(s+24.145)}$$

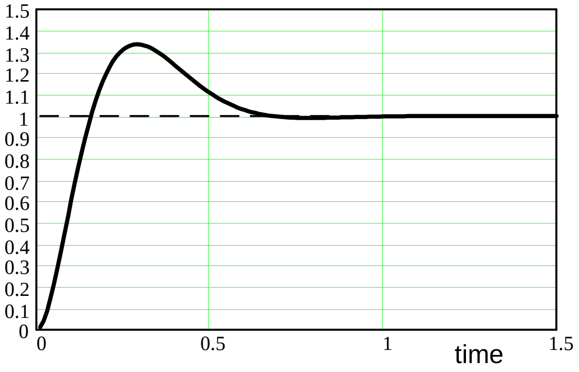
Expected overshoot

$$\zeta := \frac{45\cdot deg}{100\cdot deg} \qquad \zeta = 0.45 \qquad 100\cdot\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}} = 20.535\cdot\%$$

Root Locus with gain = 1

CL pole Locations at red squares:





% overshoot $\simeq$ $35\cdot\%$

Second case:

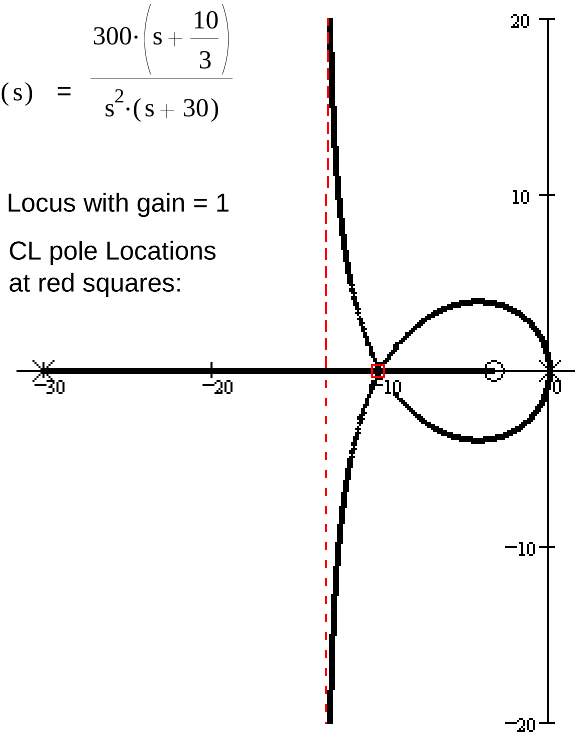
$\frac{a}{b} = a\_b := 9$ $a := \omega_c \cdot \sqrt{a\_b}$ $b := \frac{\omega_c}{\sqrt{a\_b}}$ $b = 3.333$ $k_c := \frac{\omega_c^{\ 2}}{k_p} \cdot \sqrt{a\_b}$ $k_c = 300$

$a = 30$

$$G_c(s) = \frac{300 \cdot \left(s + \frac{10}{3}\right)}{s^2 \cdot (s + 30)}$$

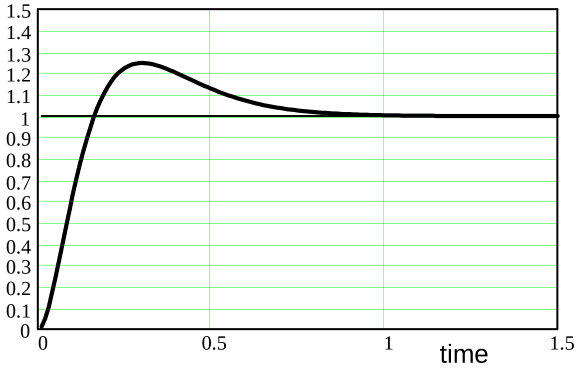
Root Locus with gain = 1

CL pole Locations
at red squares:



Expected overshoot

$\zeta := \frac{53.1 \cdot deg}{100 \cdot deg}$ $\zeta = 0.531$ $100 \cdot \% \cdot e^{\frac{-\zeta \cdot \pi}{\sqrt{1-\zeta^2}}} = 13.964 \cdot \%$



% overshoot $\simeq$ $27 \cdot \%$

Third case:

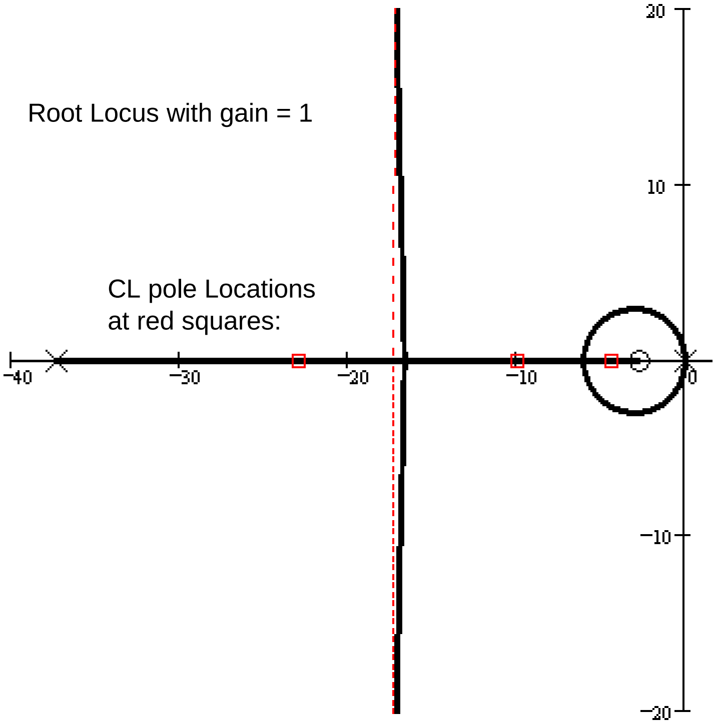
$\frac{a}{b} = a\_b := 13.9$ $a := \omega_c \cdot \sqrt{a\_b}$ $b := \frac{\omega_c}{\sqrt{a\_b}}$ $b = 2.682$ $k_c := \frac{\omega_c^{\ 2}}{k_p} \cdot \sqrt{a\_b}$ $k_c = 372.827$

$a = 37.283$

$$G_c(s) = \frac{372.827 \cdot (s + 2.682)}{s^2 \cdot (s + 37.283)}$$

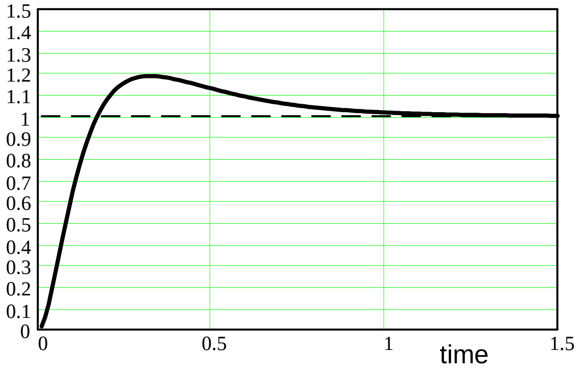
Root Locus with gain = 1

CL pole Locations
at red squares:



Expected overshoot

$\zeta := \frac{60 \cdot deg}{100 \cdot deg}$ $\zeta = 0.6$ $100 \cdot \% \cdot e^{\frac{-\zeta \cdot \pi}{\sqrt{1-\zeta^2}}} = 9.478 \cdot \%$



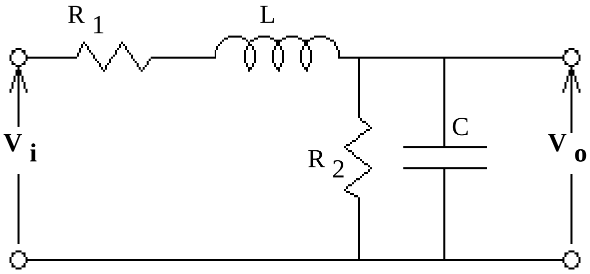
% overshoot $\simeq$ $21 \cdot \%$

Actual overshoots are much larger than expected by the table above, but, overshoots predicted by the second-order approximation are even worse (b/c of zero close to origin).

ECE 3510 Input and Output Impedance

A.Stolp 4/10/22 a

Let's find the transfer function of this circuit.



$\mathbf{H}(s)$ = ?

$$\mathbf{H}(s) = \frac{\mathbf{V}_{\mathbf{o}}(s)}{\mathbf{V}_{\mathbf{i}}(s)} = \frac{\dfrac{1}{\dfrac{1}{R_2} + C\cdot s}}{R_1 + L\cdot s + \dfrac{1}{\dfrac{1}{R_2} + C\cdot s}} \quad \frac{\dfrac{1}{R_2} + C\cdot s}{\dfrac{1}{R_2} + C\cdot s}$$

$$= \frac{1}{R_1\cdot\left(\dfrac{1}{R_2} + C\cdot s\right) + L\cdot s\cdot\left(\dfrac{1}{R_2} + C\cdot s\right) + 1}$$

$$= \frac{1}{\dfrac{R_1}{R_2} + R_1\cdot C\cdot s + \dfrac{L\cdot s}{R_2} + L\cdot s\cdot C\cdot s + 1} \quad \frac{\dfrac{1}{L\cdot C}}{\dfrac{1}{L\cdot C}} \quad = \frac{\dfrac{1}{L\cdot C}}{\dfrac{R_1}{R_2}\cdot\dfrac{1}{L\cdot C} + \dfrac{R_1\cdot C}{L\cdot C}\cdot s + \dfrac{L\cdot s}{R_2}\cdot\dfrac{1}{L\cdot C} + s^2 + \dfrac{1}{L\cdot C}}$$

$$= \frac{\dfrac{1}{L\cdot C}}{s^2 + \left(\dfrac{R_1}{L} + \dfrac{1}{R_2\cdot C}\right)\cdot s + \left(1 + \dfrac{R_1}{R_2}\right)\cdot\dfrac{1}{L\cdot C}}$$

Input Impedance

What load does this circuit place on the source of $\mathbf{V}_{\mathbf{i}}$?

$$\mathbf{Z}_{\mathbf{in}}(s) = R_1 + L\cdot s + \frac{1}{\dfrac{1}{R_2} + C\cdot s}$$

OR, if this circuit is followed by another circuit with $\mathbf{Z}_{\mathbf{in2}}(s)$, then,

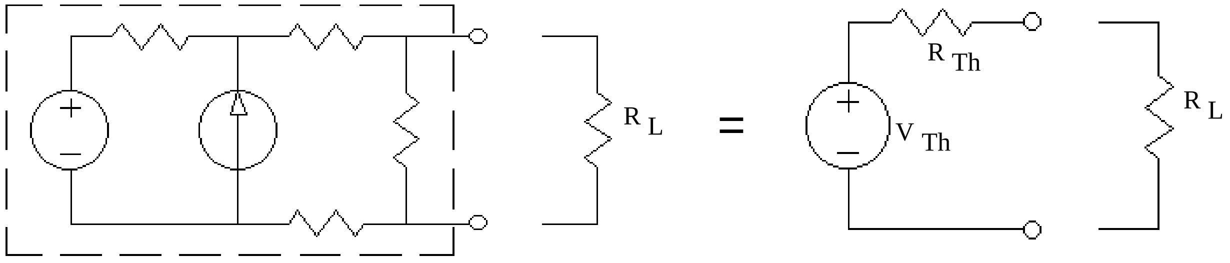
$$\mathbf{Z}_{\mathbf{in}}(s) = R_1 + L\cdot s + \frac{1}{\dfrac{1}{R_2} + C\cdot s + \dfrac{1}{\mathbf{Z}_{\mathbf{in2}}(s)}}$$

Usually, the higher the input impedance, the better.

Output Impedance

Output Impedance is just like the Thévenin Resistance

Thévevin Equivalent Circuit

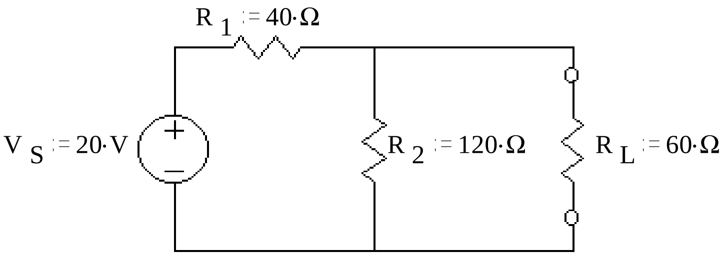


Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

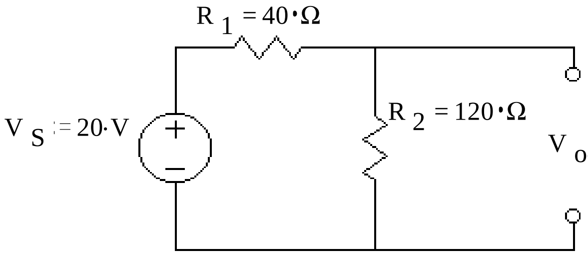
1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage (V_{Th}).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
4) Draw the Thévenin equivalent circuit and add your values.

Ex 1 Find the Thévenin equivalent:



To find a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage (V_{Th}).



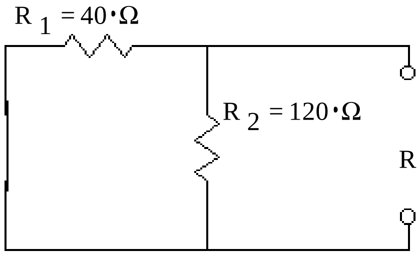
Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S \cdot \frac{R_2}{R_1 + R_2} \qquad V_{Th} = 15 \cdot V$$

2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.)
This is the Thévenin source resistance (R_{Th}).

Zero the source

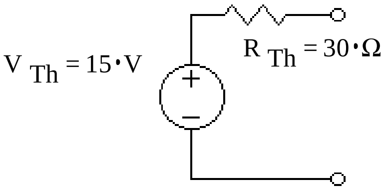


Find the Thevenin resistance:

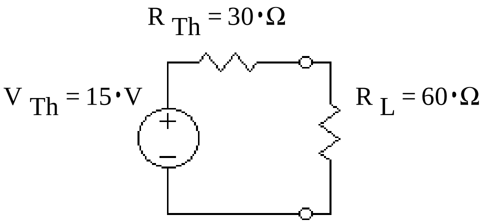
$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{Th} = 30 \cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:



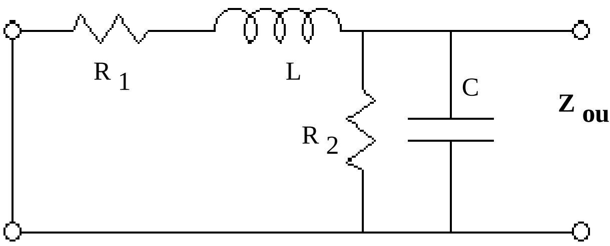
If the load were reconnected:



$$V_L = V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} = 10 \cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7 \cdot mA$$

Output Impedance



$$\mathbf{Z}_{\mathbf{out}}(s) = \frac{1}{\frac{1}{R_1 + L \cdot s} + \frac{1}{R_2} + C \cdot s} \left(\frac{R_1 + L \cdot s}{R_1 + L \cdot s}\right)$$

$$\mathbf{Z}_{\mathbf{out}}(s) = \frac{R_1 + L \cdot s}{1 + \frac{R_1}{R_2} + \frac{L}{R_2} \cdot s + C \cdot s \cdot R_1 + L \cdot C \cdot s^2} \frac{\left(\frac{1}{L \cdot C}\right)}{\left(\frac{1}{L \cdot C}\right)} = \frac{\frac{1}{C} \cdot s + \frac{R_1}{L \cdot C}}{s^2 + \left(\frac{1}{C \cdot R_2} + \frac{R_1}{L}\right) \cdot s + \frac{1}{L \cdot C} \cdot \left(1 + \frac{R_1}{R_2}\right)}$$

OR, if this circuit is preceded by another circuit with $\mathbf{Z}_{\mathbf{out0}}(s)$, then, $\mathbf{Z}_{\mathbf{out}}(s) = \frac{1}{\frac{1}{\mathbf{Z}_{\mathbf{out0}}} + \frac{1}{R_1 + L \cdot s} + \frac{1}{R_2} + C \cdot s}$

Usually, the lower the output impedance, the better

ECE 3510

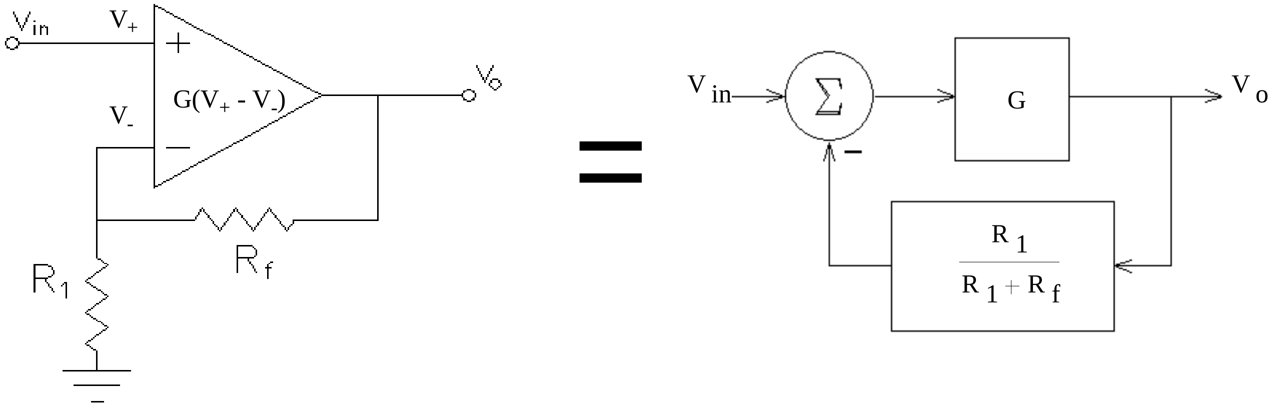
Amplifier Feedback

(Linear Amplifiers)

A. Stolp
12/5/22

Amplifier Compensation & Bandwidth Extension (Especially, Op Amps)

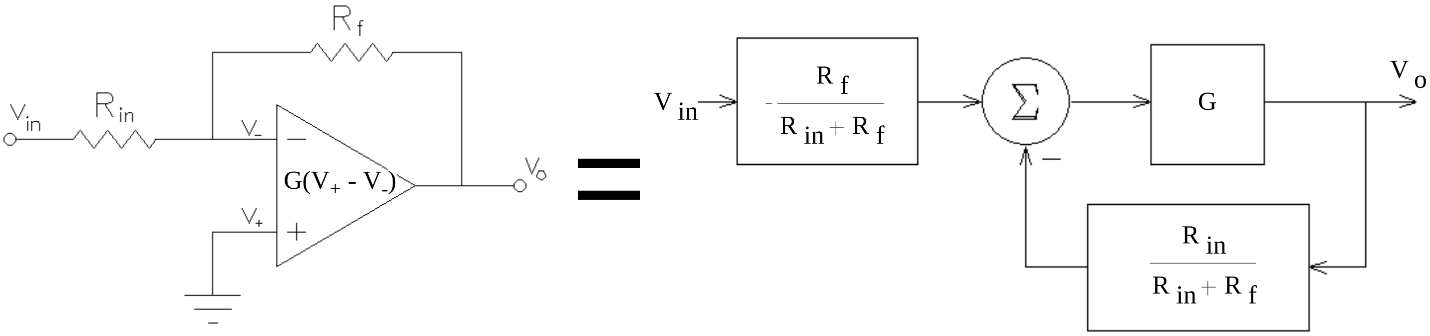
Recall from previous classes that op-amps are used with lots of negative feedback. In homework 4, you showed that common op-amp circuits could be looked at as feedback loops.



$$\frac{V_o}{V_{in}} = \frac{A}{1+A\cdot B} = \frac{G}{1+\frac{R_1}{R_1+R_f}\cdot G} = \frac{G\cdot\left(R_1+R_f\right)}{R_1+R_f+R_1\cdot G} = \frac{R_1+R_f}{\frac{R_1+R_f}{G}+R_1} \simeq \frac{R_1+R_f}{0+R_1} = 1+\frac{R_f}{R_1}$$

If G is very large.

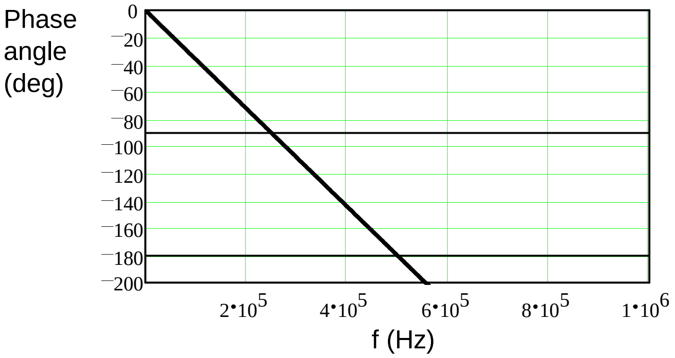
AND

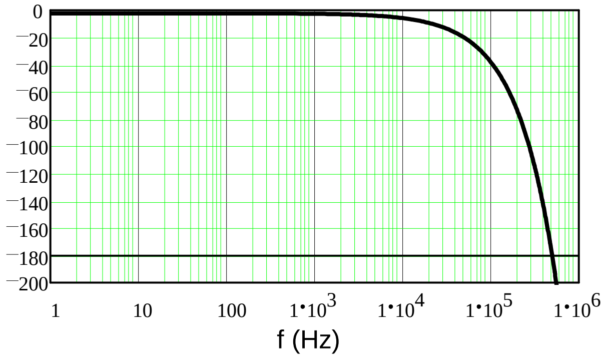


$$\frac{V_o}{V_{in}} = -\frac{R_f}{R_{in}+R_f}\cdot\frac{A}{1+A\cdot B} = -\frac{R_f}{R_{in}+R_f}\cdot\frac{G}{1+\frac{R_{in}}{R_{in}+R_f}\cdot G} = -\frac{R_f\cdot G}{R_{in}+R_f+R_{in}\cdot G} = -\frac{R_f}{\frac{R_{in}+R_f}{G}+R_{in}} \simeq -\frac{R_f}{0+R_{in}} = -\frac{R_f}{R_{in}}$$

If G is very large.

But, op-amps are made of real circuitry and therefore have a little delay. Just a delay of: $D := 1\cdot\mu s$
Would result phase angle plots like those below. Serious problems above 500kHz.





To make sure that op-amp circuits remain stable with feedback, they are compensated with a low-frequency pole so that the gain is less than 0dB where the phase angle falls off to -180° .

ECE 3510 Amplifier Feedback p2

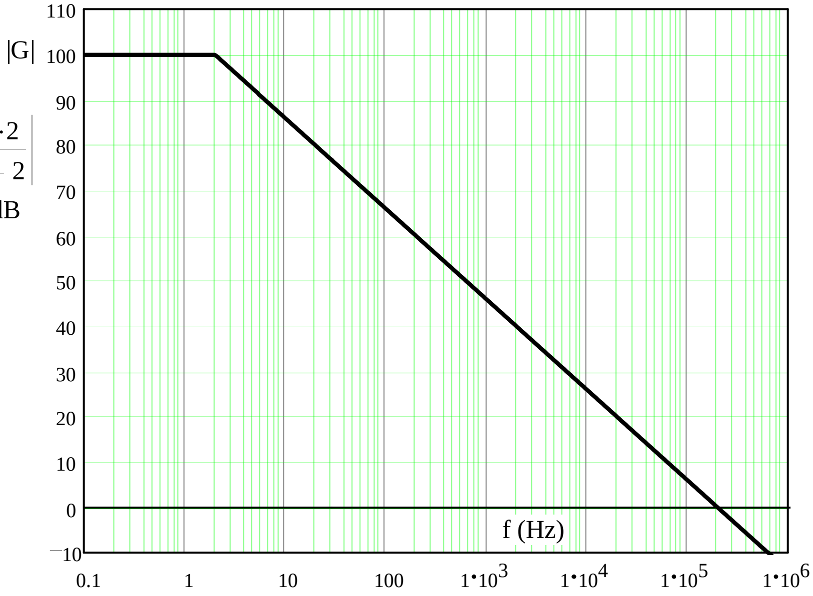
Typical op-amp compensation would look like this for an op-amp with a gain of 100k and $D = 1\mu s$.

$$G := 100000$$

Adequate compensation could be a pole at 2Hz (4π rad/sec).

Op-amp transfer function: $\dfrac{G\cdot 4\cdot\pi}{s+4\cdot\pi}$

$G\cdot 2\cdot Hz$ is the gain-bandwidth product (GBW). (in Hz)



For the noninverting amp:

$$\frac{V_o}{V_{in}} = \frac{\dfrac{G\cdot 4\cdot\pi}{s+2}}{1+\dfrac{R_1}{R_1+R_f}\cdot\dfrac{G\cdot 4\cdot\pi}{s+4\cdot\pi}}$$

$$= \frac{G\cdot 4\cdot\pi}{s+4\cdot\pi+\dfrac{R_1}{R_1+R_f}\cdot G\cdot 4\cdot\pi}$$

new pole is at: $4\cdot\pi+\dfrac{R_1}{R_1+R_f}\cdot G\cdot 4\cdot\pi \;\simeq\; \dfrac{R_1}{R_1+R_f}\cdot G\cdot 4\cdot\pi$ rad/sec

$= \dfrac{\text{GBW}}{\text{Closed-loop gain}}$ (in Hz)

For the inverting amp:

$$\frac{V_o}{V_{in}} = -\frac{R_f}{R_{in}+R_f}\cdot\frac{\dfrac{G\cdot 4\cdot\pi}{s+4\cdot\pi}}{1+\dfrac{R_{in}}{R_{in}+R_f}\cdot\dfrac{G\cdot 4\cdot\pi}{s+4\cdot\pi}} = -\frac{R_f}{R_{in}+R_f}\cdot\frac{G\cdot 4\cdot\pi}{s+4\cdot\pi+\dfrac{R_{in}}{R_{in}+R_f}\cdot G\cdot 4\cdot\pi}$$

new pole is at: $4\cdot\pi+\dfrac{R_{in}}{R_{in}+R_f}\cdot G\cdot(4\cdot\pi) \;\simeq\; \dfrac{R_{in}}{R_{in}+R_f}\cdot G\cdot(4\cdot\pi)$ rad/sec $= \dfrac{\text{GBW}}{\text{Closed-loop gain}}$ (in Hz)

The pole is also the 3dB roll-off point of the amplifier, as long as it's just a single pole.

And, The upshot is:

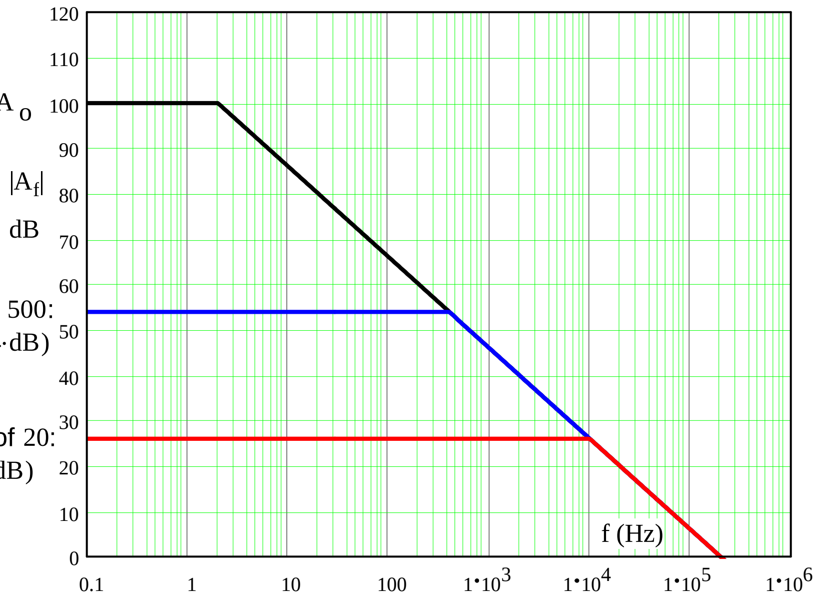
If your op-amp circuit has a closed-loop gain of "g", then the bandwidth should be about:

Closed-loop bandwidth: $\dfrac{GBW}{g}$ (in Hz)

Example closed-loop frequency responses:

for a closed-loop gain of 500: $(54\cdot dB)$

for a closed-loop gain of 20: $(26\cdot dB)$



If you want a higher frequency response, either buy a better op-amp, with a higher gain-bandwidth product, or use more stages. However, if you use two stages with the same the 3dB roll-off point, be aware that the overall roll-off will be 6dB at that frequency and that the 3dB roll-off point will be lower by about 36%.
And about 50% lower for three stages with same 3dB roll-off point.

Effects of Negative Feedback on Linear Amplifiers

In general, you trade gain for other improvements, like bandwidth.

Gain

Closed-loop gain (Gain with feedback) = $A_f = \dfrac{A_o}{1 + A_o \cdot B} = \dfrac{\text{Open-loop gain}}{\text{Amount of feedback}}$

B = the feedback factor, generally less than 1, often a lot less.

If $A_o \cdot B$ is significantly > 1, then: $A_f \simeq \dfrac{1}{B}$

This is a big difference between the control systems we have been studying and feedback used with amplifiers. Control systems generally use feedback closer to 1.

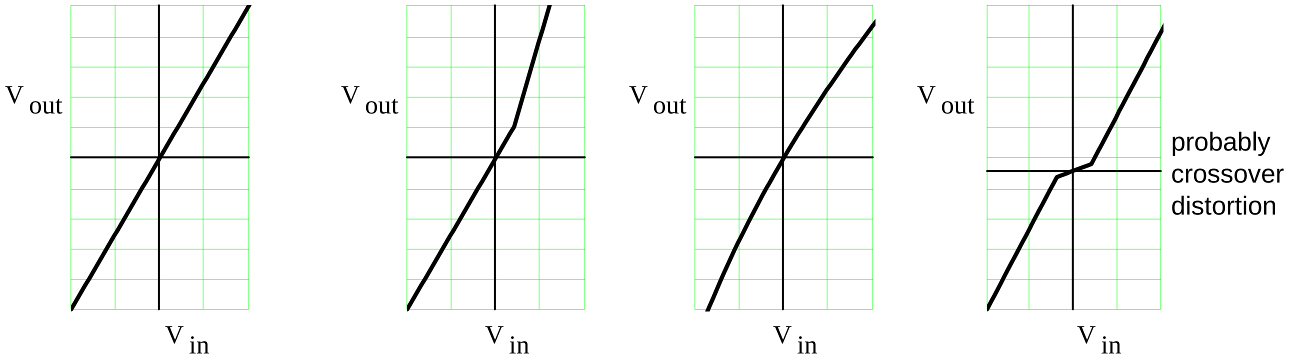
Gain is stabilized, even if the the open-loop gain (A_o) in not consistent.

Possible inconsistencies:

Gain may be be inconsistent from one device to the next due to manufacturing differences.

Gain may be be inconsistent over a range of signal frequencies, as seen above..

Gain may be be inconsistent over a range of signal amplitudes and/or output magnitudes (nonlinear gain).



Bandwidth extension

The high-frequency side of this has already been illustrated as it pertains to op amps, but it also works for low frequencies.

in general: If the open-loop amplifier has the following transfer function: $\dfrac{A_o \cdot s}{(s + \omega_L) \cdot \left(1 + \dfrac{s}{\omega_H}\right)}$

A_o = Midband gain

ω_L = Low-frequency 3dB roll-off point

ω_H = High-frequency 3dB roll-off point

With feedback

High-frequency 3dB roll-off point = $\omega_{Hf} = \omega_H \cdot (1 + A_o \cdot B)$ OR $f_{Hf} = f_H \cdot (1 + A_o \cdot B)$

Low-frequency 3dB roll-off point = $\omega_{Lf} = \dfrac{\omega_L}{1 + A_o \cdot B}$ OR $f_{Lf} = \dfrac{f_L}{1 + A_o \cdot B}$

resulting closed-loop transfer function: $\dfrac{A_f \cdot s}{(s + \omega_{Lf}) \cdot \left(1 + \dfrac{s}{\omega_{Hf}}\right)}$

A_f = Midband gain

ω_{Lf} = Low-frequency 3dB roll-off point

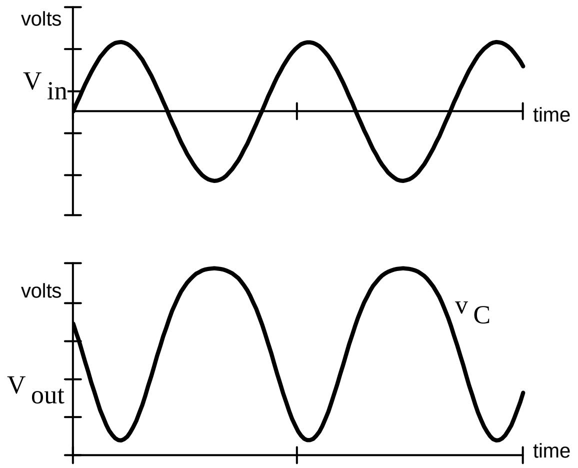
ω_{Hf} = High-frequency 3dB roll-off point

Noise Reduction (increase Signal to Noise (S/N) ratio)

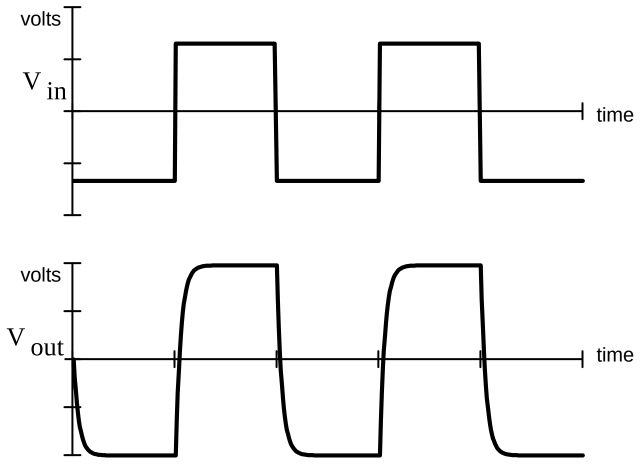
Unwanted additions to the signal, like hiss or hum, are called noise. Every amplifier stage adds some noise. Feedback can reduce noise as long as it isn't added in at the input to the first stage, where it looks just like a part of the signal. The later in the amplifier the noise is added, the more effective feedback is at reducing it.

Reduce Signal Distortion (Total Harmonic Distortion, THD)

Signal distortion is usually caused by either nonlinear gain, like those shown earlier, or by the frequency response limitations which distort the relationship of the fundamental and harmonics of a signal. Noise is also a form of distortion.

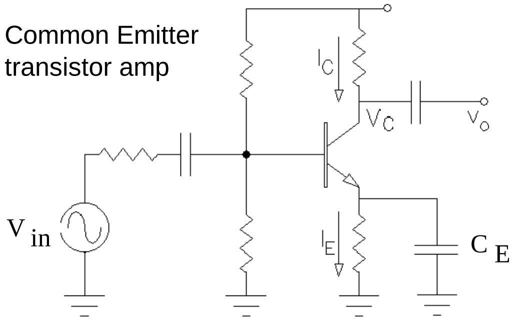


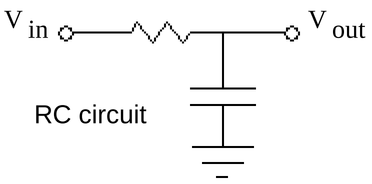
Distortion caused by nonlinear gain



Distortion caused by limited frequency response

Common Emitter transistor amp



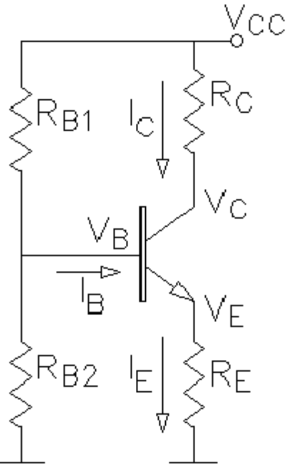


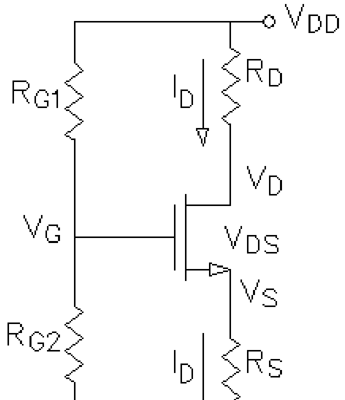
RC circuit

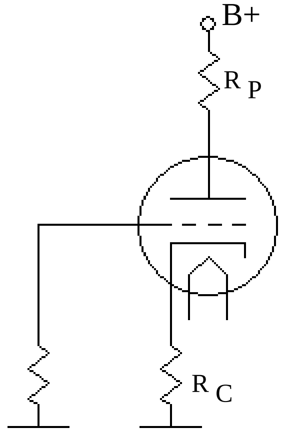
Stabilizing the gain also makes the gain more linear.
Extending the bandwidth lets more harmonics through.
Feedback improves both.

Bias Stabilization

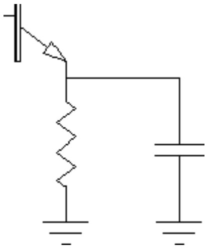
This is type of feedback is part of an individual stage of an amplifier, rather than around the entire amplifier, so, in a way, it doesn't fit the discussion here. The discrete devices we use for amplification generally need to be biased into a conduction state before a signal is applied which creates current and voltage variations about the bias points. (Exceptions are class B and class D power amplifiers.)



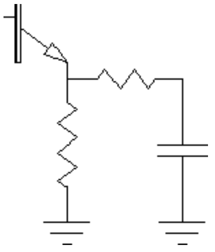




The purpose of R_E, R_S and R_C in each of these amplifier stages is the same. They provide negative feedback so as to stabilize the current flowing down though the device. The DC component of that current is the bias current. They all affect the signal as well, increasing the input impedance, reducing distortion and limiting the gain to about R_C/R_E, R_D/R_S, and R_P/R_C respectively. Placing a bypass capacitor in parallel with this resistor can remove the feedback at signal frequencies, but you loose the other benefits. Partial bypass can be a compromise.



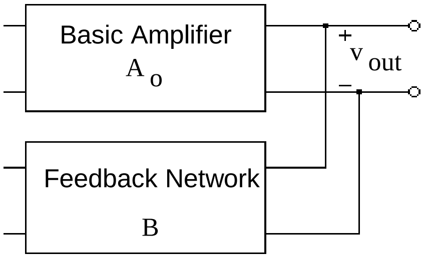
Bypass



Partial bypass

Output Impedance

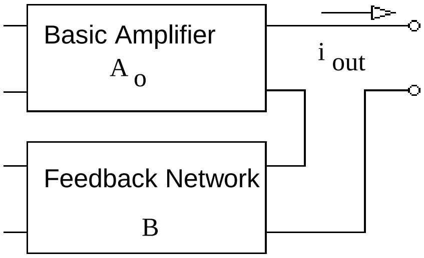
For an amplifier with voltage feedback, the output impedance is reduced. By "voltage feedback", I mean that the feedback signal is a sample of the output voltage. Also called "shunt" feedback. (See the four feedback topologies.)



Ideally: $Z_{outf} = \dfrac{Z_{out}}{1 + A_o \cdot B}$

If the desired output of your amp is voltage, then a lower output impedance is exactly what you want.

For an amplifier with current feedback, the output impedance is increased. By "current feedback", I mean that the feedback signal is a sample of the output current. Also called "series" feedback. (See the four feedback topologies.)



Ideally: $Z_{outf} = Z_{out} \cdot \left(1 + A_o \cdot B\right)$

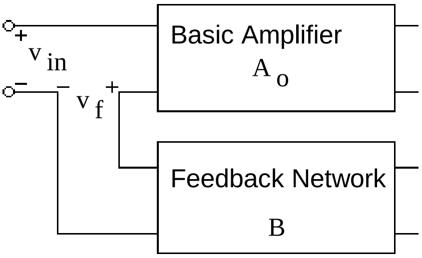
If the desired output of your amp is current, then a higher output impedance is exactly what you want.

Input Impedance

For an amplifier with voltage mixing, the input impedance is increased. By "voltage mixing", I mean that the feedback signal is voltage combined in series with the input voltage. Also called "series" mixing. (See the four feedback topologies.)

Ideally: $Z_{in} \cdot \left(1 + A_o \cdot B\right) = Z_{inf}$

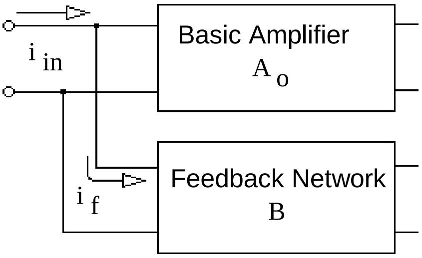
If the desired input of your amp is voltage, then a higher input impedance is exactly what you want.



For an amplifier with current mixing, the input impedance is reduced. By "current mixing", I mean that the feedback signal is a current reducing the input current. Also called "shunt" mixing. (See the four feedback topologies.)

Ideally: $\dfrac{Z_{in}}{1 + A_o \cdot B} = Z_{inf}$

If the desired input of your amp is current, then a lower input impedance is exactly what you want.



Assumptions to get ideal answers:

1. Feedback networks have ideal input and output resistances.
2. Loads are ideal for the type of output. $R_L = \infty$ for voltage outputs. $R_L = 0$ for current outputs.

Otherwise improvements in output and input impedances will be less than ideal.

To Find the Input Impedance

1. Draw out the full innards of all the boxes in the appropriate topology shown below, including all the non-ideal characteristics you want to include.
2. For topologies a) and d), below, connect nothing to the output, unless you want to see the the effect of the load on the input impedance. Even then, it's easier to work it out without a load first and then add the effect of the load in later.

   For topologies b) and c), below, connect a short to the output, unless you want to see the the effect of the load. Again, it's easier to work without a load first.

3. For topologies a) and c), below, connect an ideal voltage source to the input and find the input current. Use whatever circuit analysis methods you like.

   For topologies b) and d), below, connect an ideal current source to the input and find the input voltage. Use whatever circuit analysis methods you like.

4. $$Z_{inf} = \frac{V_{in}}{i_{in}}$$

To Find the Output Impedance

1. Draw out the full innards of all the boxes in the appropriate topology shown below, including all the non-ideal characteristics you want to include.
2. For topologies a) and c), below, connect a short at the input.

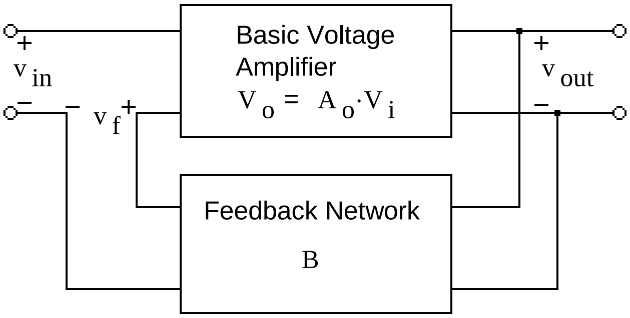
   For topologies b) and d), below, connect nothing to the input.

3. For topologies a) and d), below, connect an ideal voltage source to the output and find the current flowing back into the output.

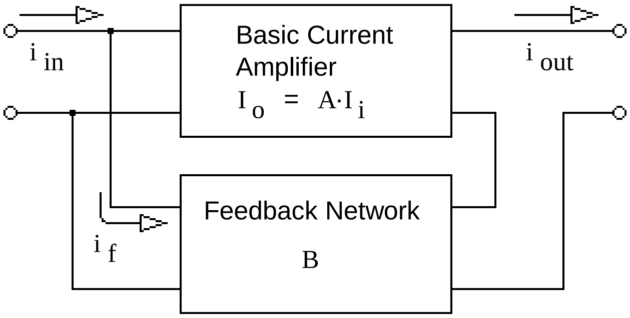
   For topologies b) and c), below, connect an ideal current source to the output to force current back into the output. Find the voltage at the output.

4. $$Z_{inf} = \frac{V_{out}}{-i_{out}}$$

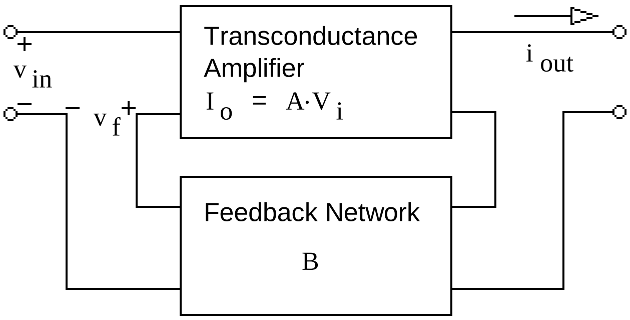
The Four Feedback Topologies



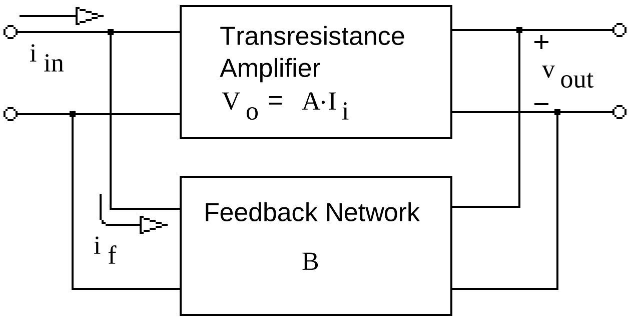
a) Voltage-mixing voltage-sampling (series-shunt)



b) Current-mixing current-sampling (shunt-series)



c) Voltage-mixing current-sampling (series-series)



d) Current-mixing voltage-sampling (shunt-shunt)

Some examples of the different topologies

a) Voltage-mixing voltage-sampling (series-shunt)

R_e is the AC signal resistance from emitter to ground R_E is the DC resistance from emitter to ground

Input impedance: $R_i = R_{B1} \| R_{B2} \| \beta \cdot (r_e + R_e)$

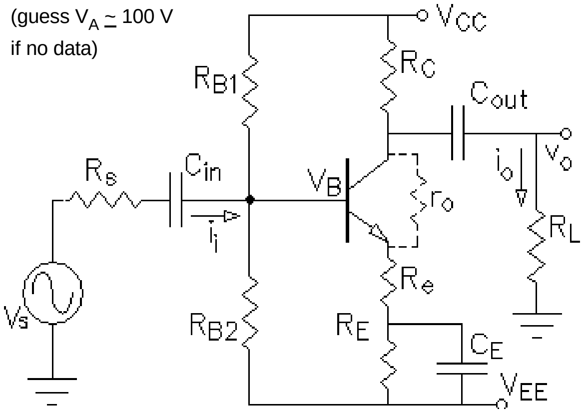
Output impedance: $R_o = R_C \| r_o$ Often neglected $r_o = \frac{V_A}{I_C}$ Early voltage. (guess $V_A \simeq 100$ V if no data)

AC collector resistance: $r_c = R_C \| R_L \| r_o$

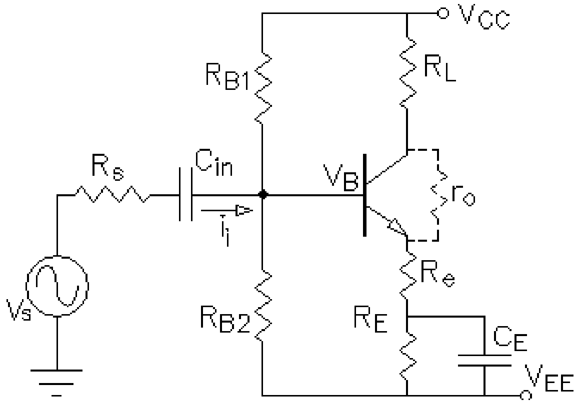
Voltage gain: $A_v = \frac{v_o}{v_b} = \frac{r_c}{r_e + R_e}$

OR: $\frac{v_o}{v_s} = \frac{R_i}{R_S + R_i} \cdot \frac{r_c}{r_e + R_e}$

Current gain: $A_i = \frac{i_o}{i_i} = \frac{r_c}{r_e + R_e} \cdot \frac{R_i}{R_L} = A_v \cdot \frac{R_i}{R_L}$



c) Voltage-mixing current-sampling (series-series)

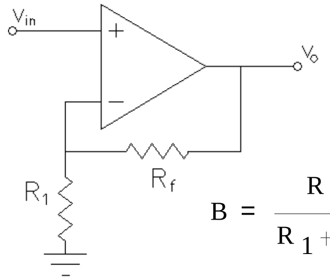


Input impedance: $R_i = R_{B1} \| R_{B2} \| \beta \cdot (r_e + R_e)$

Output impedance: r_o $r_o = \frac{V_A}{I_C}$ Early voltage. (guess $V_A \simeq 100$ V if no data)

Transconductance $g = \frac{1}{r_e + R_e}$

a) ~~Voltage-mixing~~ voltage-sampling (~~series~~-shunt)



$B = \frac{R_1}{R_1 + R_f}$

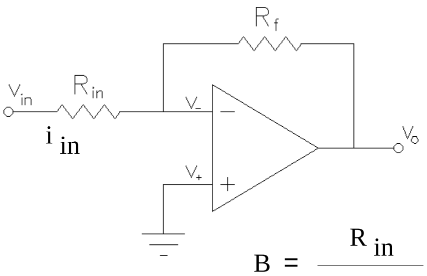
Input impedance: see op-amp data for R_i

Output impedance: $R_{outf} = \frac{R_{out}}{B}$ up to current limits of the op amp

Voltage gain: $1 + \frac{R_f}{R_1}$

Feedback signal mixing is done within the op amp.

d) Current-mixing voltage-sampling (shunt-shunt)



$B = \frac{R_{in}}{R_{in} + R_f}$

Input impedance: R_{in} for voltage input

Input impedance: $\simeq$ 0 for current input if you remove R_{in}

Output impedance: $R_{outf} = \frac{R_{out}}{B}$ up to current limits of the op amp

Voltage gain: $-\frac{R_f}{R_1}$