ECE 3510 Exam 2 given: Fall 22

1. (22 pts) This system: $\mathbf{H}(s) = \dfrac{s+12}{s+6}$ Has this input: $x(t) = 3\cdot\sin(10\cdot t)\cdot u(t)$

   a) Express the output, and separate into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

   b) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

   c) Use steady-state AC analysis to find the phasor representation of the steady-state output in polar form.

   $\mathbf{Y}_{ss}(j\omega) = ?$

   d) Express the complete (both transient and steady-state) output as a function of time. $y(t) = ?$

   e) What is the time constant of the transient part this expression? $\tau = ?$

2. (2 pts) Our time-domain signals all start at time = 0 because we use the unilateral Laplace transform to represent the signals. How are conditions and events that happened before time = 0 handled?

3. (10 pts) Several transfer functions are shown below. Without doing anything more than looking at the transfer function, try to determine if it is BIBO stable. Answer Y, N, or C for each.

Y) definitely BIBO table

N) definitely NOT BIBO table

C) can't tell just by looking

a) $\dfrac{s\cdot(s+6)}{s^5+4\cdot s^4+2\cdot s^3+6\cdot s^2+2\cdot s+1}$ ______

b) $\dfrac{s+1}{s^5+3\cdot s^4-18\cdot s^3+3\cdot s^2+s+2}$ ______

c) $\dfrac{s-4}{s^4+2\cdot s^3+1\cdot s^2+s+0.25}$ ______

d) $\dfrac{s^4+10\cdot s^2+s}{\left(s^2+2\cdot s+5\right)\cdot\left(s^2+4\cdot s+4\right)}$ ______

e) $\dfrac{10\cdot s+1}{(s+2)\cdot\left(s^2+2\cdot s+4\right)\cdot s}$ ______

f) $\dfrac{s+2}{5\cdot s^6+13\cdot s^4+8\cdot s^3+s^2+s+2}$ ______

____ / 10

4. (12 pts) The controller and plant transfer functions shown below are part of a standard unity feedback system.

$$\mathbf{C}(s) = \frac{1}{s+3} \qquad \mathbf{P}(s) = \frac{2s+8}{s-2}$$

a) As is, without any extra gain in the loop, will the whole feedback system be BIBO stable? You must justify your answer.

b) If you added gain factor to the controller, so that it is now: $C(s) = \dfrac{k}{s+3}$ Can you now change the stability of the system?
(That is, make stable if it was unstable, or unstable if it was stable.)
You must justify your answer and find the k value to make the change, if possible.

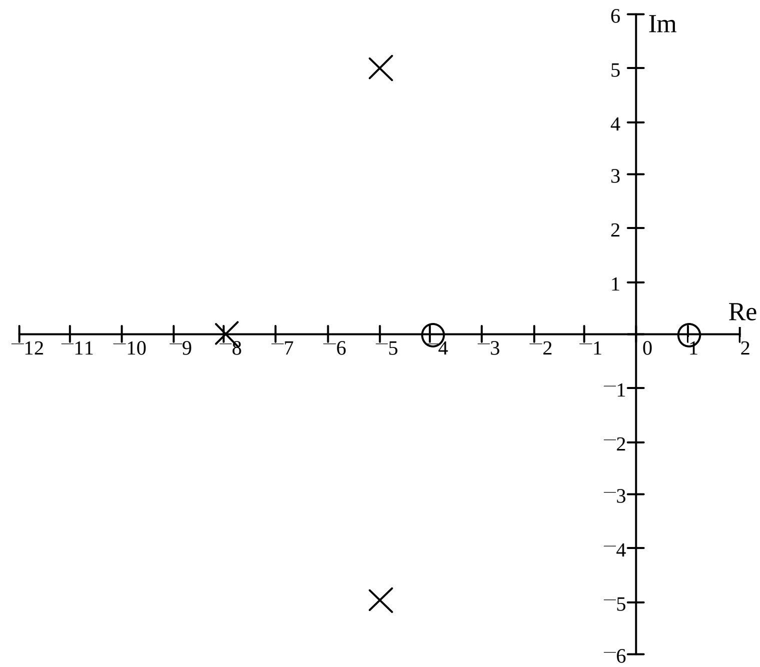
5. (16 pts) Sketch the root-locus plots for the poles and zeroes:
   Use only the rules you were told to memorize, that is, you may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid). Draw things like the asymptote angles carefully.

   a) sketch

   b) Find the open-loop transfer function.

   $\mathbf{G}(s) = ?$

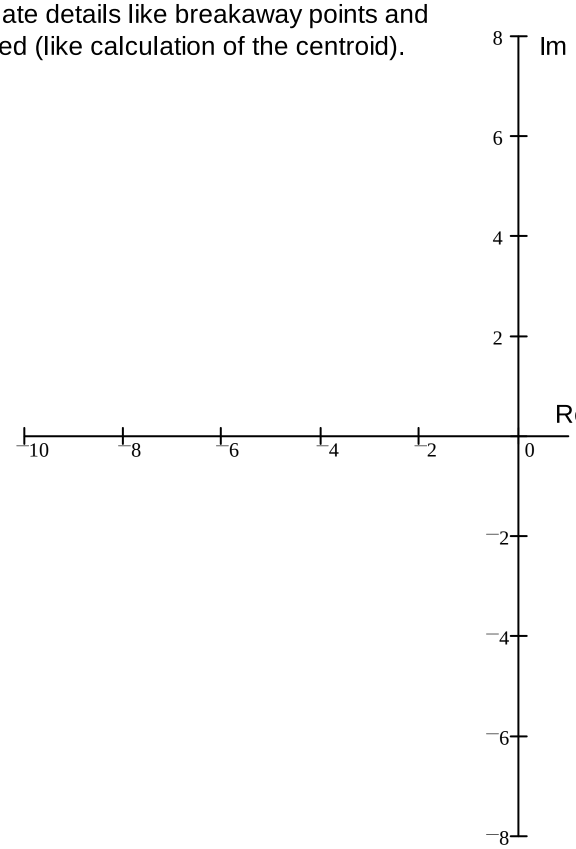
   c) Find the range of gain (k) for which the system is closed-loop stable. Assume $k > 0$.



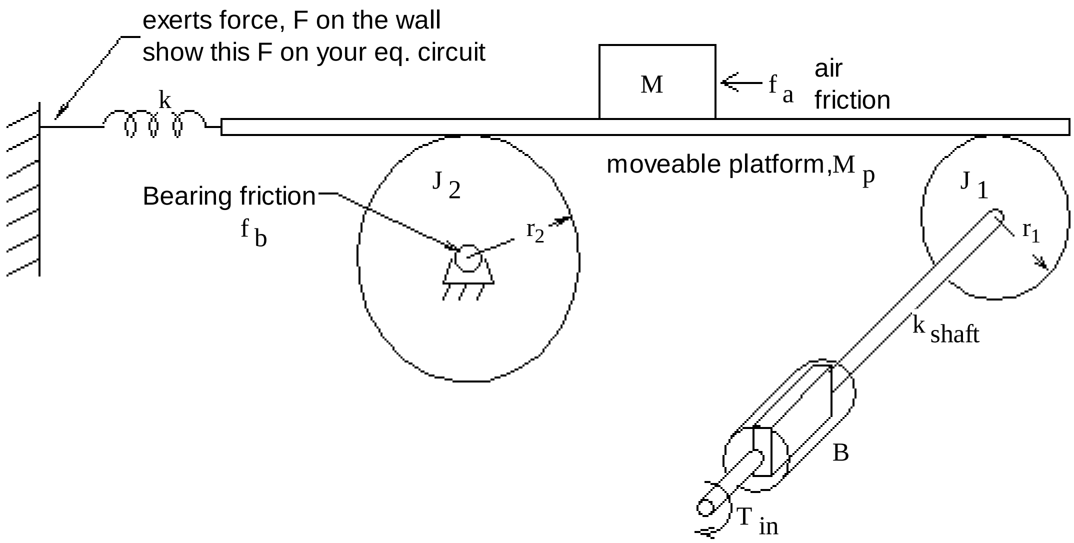
6. (12 pts) Sketch the root-locus plots for the following open-loop transfer functions:
   Use only the rules you were told to memorize, that is, you may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid). Draw things like the asymptote angles carefully.

   a) sketch

$$\mathbf{G}(s) = \frac{1}{\left(s^2 + 6\cdot s + 25\right)\cdot(s+6)\cdot(s+8)}$$



7. (26 pts) Find the equivalent electric circuit for the mechanical system shown. It is a moveable platform of mass, M_p, which rests on 2 different wheels. The input to the system is a torque, T_{in}.



a) Show the circuit with one or more transformers. Show the parts in terms of M's, k's, B's, etc., above. Indicate the force exerted on the wall, F, on your drawing.

b) Show how to eliminate a transformer (choose the easiest), just like you did in the homework. Show the equivalent parts in terms of M's, k's, J's, etc., above. You don't have to redraw the whole circuit as long as I can tell how the section of the circuit you draw would connect above.

Answers

1. a) $\frac{A}{s+6}+\frac{B\cdot s}{\left(s^2+100\right)}+\frac{C\cdot 10}{\left(s^2+100\right)}$ b) 1.324 c) $4.017 \;\angle\; -109.2\cdot\deg$

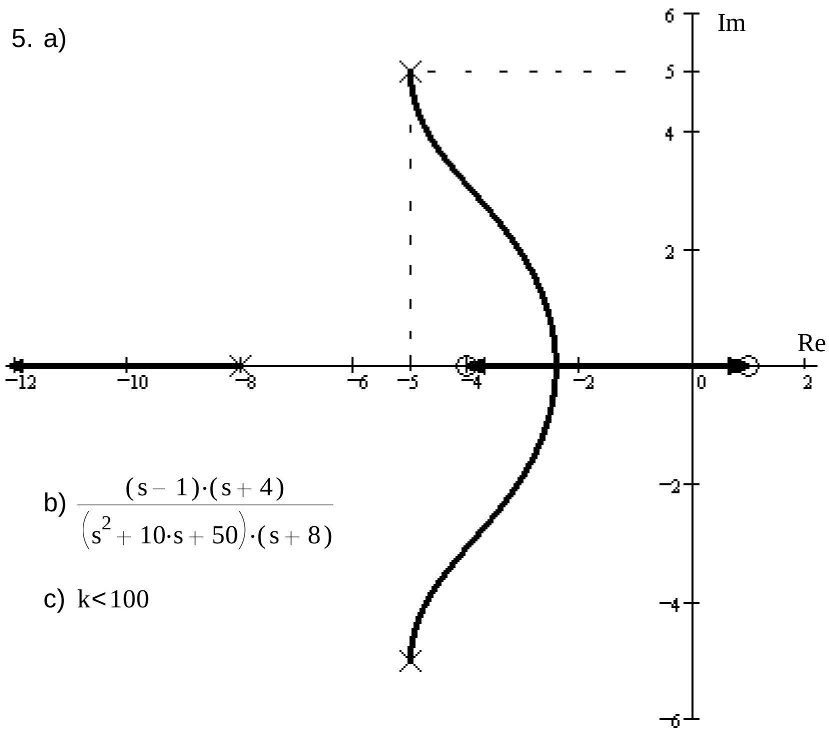
   d) $\left(1.324\cdot e^{-6\cdot t}+4.017\cdot\cos(10\cdot t-109.2\cdot\deg)\right)\cdot u(t)$ e) $\frac{1}{6}$

2. As initial conditions and/or as the initial state of the system.

3. C N C Y N N

4. a) YES b) Becomes unstable for $k \le 3/4$

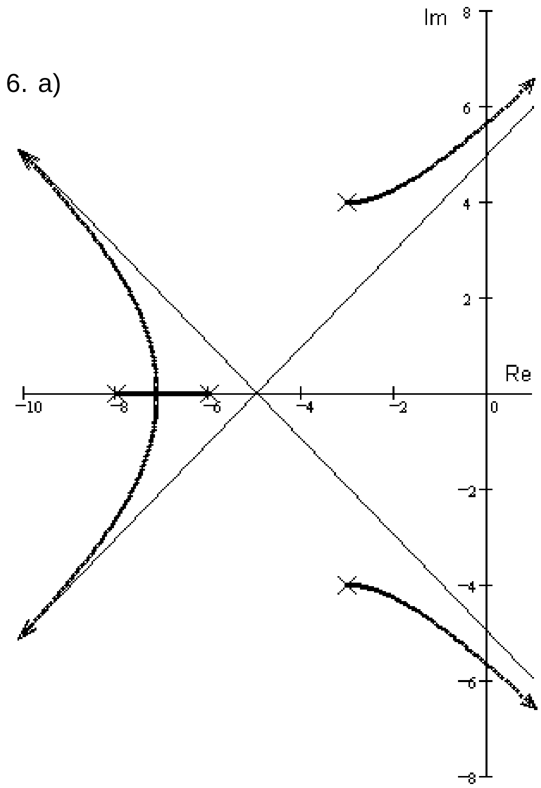
5. a)



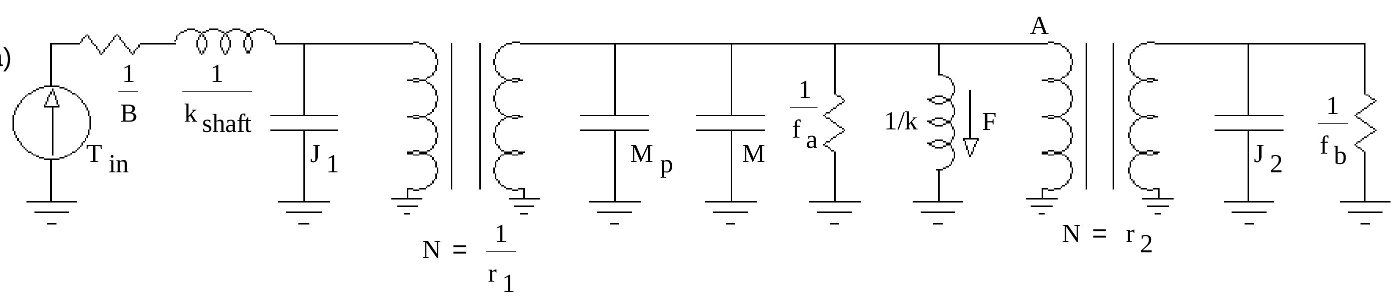
b) $\frac{(s-1)\cdot(s+4)}{\left(s^2+10\cdot s+50\right)\cdot(s+8)}$

c) $k<100$

6. a)



7. a)



b)