This part of the exam is Closed book, Closed notes, No Calculator.

1. ( 2 pts ) Our time-domain signals all start at time $=0$ because we use the unilateral Laplace transform to represent the signals. How are conditions and events that happened before time $=0$ handled?
2. ( 5 pts ) The output of a system is given by:

$$
Y(s)=\frac{\mathrm{b}_{2} \cdot \mathrm{~s}^{2}+\mathrm{b}_{1} \cdot \mathrm{~s}+\mathrm{b}_{0}}{\mathrm{~s}^{2}+\mathrm{a}_{1} \cdot \mathrm{~s}+\mathrm{a}_{0}} \cdot X(\mathrm{~s})+\frac{\mathrm{s} \cdot \mathrm{y}(0)+\frac{\mathrm{d}}{\mathrm{dt}} \mathrm{y}(0)+\mathrm{a}_{1} \cdot \mathrm{y}(0)-\mathrm{b}_{2} \cdot \mathrm{~s} \cdot \mathrm{x}(0)-\mathrm{b}_{2} \cdot \frac{\mathrm{~d}}{\mathrm{dt}} \mathrm{x}(0)-\mathrm{b}_{1} \cdot \mathrm{x}(0)}{\mathrm{s}^{2}+\mathrm{a}_{1} \cdot \mathrm{~s}+\mathrm{a}_{0}}
$$

a) What is $y(0)$ ? Or, what does it mean?
b) What is $\frac{\mathrm{d}}{\mathrm{dt}} \mathrm{y}(0)$ ? Or, what does it mean?
3. ( 6 pts) The time constant of a system's response must better than 0.25 sec and the damping factor must be greater than 0.707 . Does that mean the system's poles must lie in a certain region of the s-plane? If yes, show that area on drawing at right, including numbers where appropriate. Make it clear where the poles must lie. If no, write NO below.

4. (7 pts) In Lab 4 you characterized a DC motor. The curve at right is a typical rotational speed vs time measurement for such a motor. Tell me what motor parameter or characteristic is most responsible for each of the labeled parts of the curve.
A, first curve
$B$, second curve
C, final value

4. (7 pts) a) What should a feedback system have so that it will perfectly reject constant disturbances AND perfectly track constant inputs?
b) Say the same thing in another way. (Time-domain instead of frequency domain OR vice versa.)
c) How or why does this work?
5. ( 6 pts )
a) Can you practically and effectively cancel a pole in the RHP with a zero?
b) Can you practically and effectively cancel a pole in the LHP with a zero?
6. (12 pts) a) Sketch the root-locus plots for the open-loop poles and zeros shown. Show your work where needed. (Like calculation of the centroid, but NOT breakaway points or departure angles.).
b) Find the range of gain (k) for which the system is closed-loop stable. Assume $\mathrm{k}>0$.

Why or why not?
Why or why not?

1. (20 pts) Find the equivalent electric circuit for the system shown. $\mathrm{V}_{\text {in }}$ is a voltage input to a standard DC motor with all the elements $\left(\mathrm{R}_{\mathrm{a}}, \mathrm{L}_{\mathrm{a}}, \mathrm{K}, \mathrm{B}_{\mathrm{m}}, \& \mathrm{~J}_{\mathrm{m}}\right)$. It is connected to a conveyor belt by a shaft which has some flex. The pulleys each have some inertia ( $\mathrm{J}_{\mathrm{p}}$ ) and friction $\left(\mathrm{f}_{\mathrm{p}}\right)$. The belt has some flex and moves a mass, M. There is a viscous friction, $f_{B M}$ between the belt and the mass.

a) Show the circuit with transformer(s). Show the parts in terms of J's, k's, b's, etc., above, including the "turns ratios". Use room on the next page if you need to.
b) Indicate the point in the circuit which represents the velocity of the mass, M. Is it the voltage or the current which represents the velocity?
c) Redraw the last 4 or 5 components of the circuit, but show that part circuit without a transformer, just like you did in the homework. Show the equivalent parts in terms of J's, k's, b's, etc., above.
2. (15 pts) A plant with the transfer function shown below is part of a standard unity feedback system with gain k .
a) Use the Routh-Hurwitz method to determine the stability of the whole feedback system.
b) Does $k$ play a role in the stability? If yes, determine the value range of $k$ that will produce a stable system.

$$
G(s)=\frac{2 \cdot s^{2}-3}{s^{3}+5 \cdot s^{2}+2 \cdot s+9}
$$

DO NOT assume $k>0$
3. (20 pts) a) Sketch the root-locus for the open-loop transfer function below.

$$
\mathrm{G}(\mathrm{~s})=\frac{(\mathrm{s}+6)}{(\mathrm{s}+1)^{2} \cdot\left[(\mathrm{~s}+5)^{2}+3^{2}\right]}
$$

b) Find the departure angle from the positive complex pole.
c) Does the root locus cross the imaginary axis at $4 j$ ? Show your evidence.

## Answers

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1. As initial conditions and/or as the initial state of the system.
2. a) The initial value of the output
b) The initial slope of the output
3. $\mathrm{A} \mathrm{L}_{\mathrm{a}}$, The armature inductance

B $\mathrm{J}_{\mathrm{m}}$, The motor's moment of inertia or The mechanical inertia
C $\mathrm{K}_{\mathrm{V}}$, The motor's generation constant
5. a) The controller, $\mathrm{C}(\mathrm{s})$, should have a pole at the origin.
b) The controller, C, should have a an integrator.
3.

c) Time-domain answer:

The integrator makes even a very small DC error big enough to change the output and eliminate the error.
Freq.-domain answer:
The pole at zero makes the DC-gain of the error signal zero relative to both the input and disturbances.
6. a) NO The zero will never exactly match the pole, so there will always be a branch of the root-locus in the RHP and that closed-loop pole will always be unstable.
b) YES Although the zero will never exactly match the pole, it doesn't matter in the LHP.
The root locus may stop and start where it would otherwise be continuous, it may loop around either the pole or zero or it may have a separate branch from the pole to the zero. In all three cases the remaining closed-loop pole will always be very close to a zero and thus have very little affect on the output.

b) $\mathrm{k}<6.5$

## Open-book part




2. a) $\mathrm{s}^{3}$ |  |  |
| :---: | :---: |
| $\mathrm{s}^{3}$ | 1 |
| $\mathrm{~s}^{2}$ | $(5+2 \cdot \mathrm{k})$ |
| $\mathrm{s}^{1}$ | $\frac{2 \cdot(5+2 \cdot \mathrm{k})-(9-3 \cdot \mathrm{k})}{(5+2 \cdot \mathrm{k})}$ |
|  | $\begin{array}{c}-1\end{array}$ |
| $\mathrm{~s}^{0}$ | $(9-3 \cdot \mathrm{k})$ |
3. a) at right
b) $-124.7 \cdot \operatorname{deg}$
c) $\mathrm{s}:=4 \cdot \mathrm{j}$
$\arg \left[\frac{(s+6)}{(s+1)^{2} \cdot\left[(s+5)^{2}+3^{2}\right]}\right]=175.99 \cdot \operatorname{deg} \quad \mathrm{NO}$

$$
\left\{\frac{\mathrm{r}^{2}}{\mathrm{f}_{\mathrm{b}}}\right.
$$

b) YES $-\frac{1}{7}<\mathrm{k}<3$
$(9-3 \cdot k) \quad 0$
$0 \quad 0$

$$
\mathrm{N}=\frac{1}{\mathrm{r}}
$$

b) voltage at black point above represents the velocity of the mass.

