## ECE 3510 Exam 2 given: Spring 23 <br> (Some of the space between problems has been removed.)

This part of the exam is Closed book, Closed notes, No Calculator.
Sketch the root-locus plots for the following open-loop transfer functions:
Use only the rules you were told to memorize, that is, you may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid). Draw things like the asymptote angles carefully.

1. (11 pts)
a) sketch

b) Find the range of gain (k) for which the system is closed-loop stable. Assume $\mathrm{k}>0$.

Remember, I asked for a range for stability
2. (11 pts) $G(s)=\frac{1}{\left(s^{2}+2 \cdot s+10\right) \cdot(s+4) \cdot(s+6)}$


1. (12 pts) The controller and plant transfer functions shown below are part of a standard unity feedback system.

$$
C(s)=\frac{1}{s+5}
$$

$$
\mathrm{P}(\mathrm{~s})=\frac{2 \mathrm{~s}+3}{\mathrm{~s}-3}
$$

a) As is, without any extra gain in the loop, will the whole feedback system be BIBO stable? You must justify your answer.
b) If you added gain factor to the controller, so that it is now: $C(s)=\frac{k}{s+5}$ Can you now change
the stability of the system?
(That is, make stable if it was unstable, or unstable if it was stable.)
You must justify your answer and find the k value to make the change, if possible.
2. ( 10 pts ) a) Determine if the break-away point is at -4.5 . Show your evidence. I want to see specific calculations and numbers to justify your answer.

b) The gain required to place a closed loop pole at -4.5 is: Answer without making more calculations.
A) LESS than the gain required to place the closed loop poles at the break-away point.
B) THE SAME as the gain required to place the closed loop poles at the break-away point.
C) GREATER than the gain required to place the closed loop poles at the break-away point.
D) It isn't possible to answer this without more calculations.
3. (30 pts) a) Sketch (or use a dotted line) the root ECE 3510 Exam 2 Spring 23 p4 locus for the OL transfer function shown below.
$\mathbf{G}(\mathrm{s})=\frac{\mathrm{s}+9}{(\mathrm{~s}+1) \cdot\left(\mathrm{s}^{2}+4 \cdot \mathrm{~s}+8\right)}$

b) Does the root locus cross the j $\omega$ axis at 5?

Be sure to show the work and method you used to decide.
c) Regardless of what you found in part c), continue to assume that the root-locus crosses the $\mathrm{j} \omega$ axis at 5 . Give the range of gain k for which the system is closed-loop stable.

Remember, I asked for a range for stability
d) Find the departure angle from one of the complex poles.

It may be helpful redraw the open-loop poles and zero without lines.

e) Use what you found in parts b) and c) to draw your final root-locus plot, above.

Clearly show the angle and possibly the crossing (show numbers on the drawing).
4. (26 pts) Find the equivalent electric circuit for the mechanical systeI

a) Show the circuit with a transformer. Show the parts in terms of M's, k's, B's, etc., above. Indicate $\mathrm{V}_{\mathrm{M} 1}$ on your drawing.
b) Show how to eliminate the transformer, just like you did in the homework. Show the equivalent parts in terms of M's, k's, J's, etc., above. You don't have to redraw the whole circuit as long as I can tell how the section of the circuit you draw would connect above.
$\qquad$
$\qquad$
$\qquad$ / 100

## Answers


b) denominator of the closed-loop system $=s^{2}+(2+2 \cdot k) \cdot s+(3 \cdot k-15)$

Becomes stable for $k>5$
2. a) $\frac{1}{-4.5+2}=-0.4 \stackrel{?}{=} \frac{2}{-4.5+3}+\frac{1}{-4.5+6}=-0.667$
b) A)

b) $29-56.3-78.7-74=-180$
b) $\begin{aligned} & 29-56.3-78.7-74=-180 \\ & \text { YES }\end{aligned}$
c) $\mathrm{k}<13$
d) $-10.6 \cdot \mathrm{deg}$
e)

4. a)

$M_{3} \cdot\left(\frac{r_{2}}{r_{1}}\right)^{2}=\frac{Q^{-}}{\square} \frac{1}{k_{3}} \cdot\left(\frac{r_{1}}{r_{2}}\right)^{2}$

