# ECE 3510 Exam 3 Information

This sheet and the Information sheets from Exams 1 & 2 are the only reference materials allowed at exam. Bring this page. You **may add** whatever you want to this sheet (both sides).

## To create an unconventional root locus plot:

1. Determine the gain factor if it can be adjusted, and make it part of the open-loop transfer function, $G(s)$. Hold it constant at some number.
2. Determine the denominator of the closed-loop transfer function, $H(s)$. Let's call it $D_H(s)$.
3. Rearrange $D_H(s)$ into this form: $D'(s) + x \cdot N'(s)$ where $x$ is the variable for which you want to draw the root locus. Notice that $x$ occupies exactly the same position the gain would normally occupy. Normal: $D_G(s) + k \cdot N_G(s)$
   Note: If you cannot rearrange $D_H(s)$ into this form, then you cannot use this method to create an root locus plot for the variable $x$. Now: $D'(s) + x \cdot N'(s)$
4. Now simply draw a root locus as though $D'(s)$ was the open-loop denominator and $N'(s)$ was the open-loop numerator.

## Root Locus Design

**PI** To eliminate steady-state error (for constant inputs) & perfect rejection of constant disturbances
Add pole at 0 and zero at close by

**LAG** An alternative is a Lag Compensator, with a pole near the origin and a zero a little farther away.

**PD or PID** To Improve the dynamic response, add a zero to affect angles.

**LEAD** An alternative to the differentiator is a Lead Compensator with a zero and a pole much farther left.

## Root Locus Design Crib Sheet

Using 2nd-order approximation: $\dfrac{N(s)}{(s+a)^2 + b^2} = \dfrac{N(s)}{s^2 + 2 \cdot a \cdot s + a^2 + b^2} = \dfrac{N(s)}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2}$

$\omega_n^2 = a^2 + b^2$ $\omega_n$ = natural frequency $\zeta \cdot \omega_n = a$

$\zeta = \dfrac{a}{\omega_n} = \dfrac{a}{\sqrt{a^2 + b^2}}$ = damping factor $\zeta = \sin\left(\operatorname{atan}\left(\dfrac{a}{b}\right)\right)$

Overshoot: $OS = e^{-\pi \cdot \frac{a}{b}}$ $\%OS = 100\% \cdot e^{-\pi \cdot \frac{a}{b}}$ $\dfrac{a}{b} = \dfrac{\ln(OS)}{-\pi}$

2% settling time: $T_s = \dfrac{4}{a} = \dfrac{4}{\zeta \cdot \omega_n}$ $\%OS = 100\% \cdot e^{\frac{-\zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$ $\zeta = \dfrac{-\ln(OS)}{\sqrt{\pi^2 + (\ln(OS))^2}}$

Weird forms from Nise book

Static error constant (position): $K_p = \lim_{s \to 0} K \cdot C(s) \cdot G(s)$ $e_{step}(\infty) = e_{step} = \dfrac{1}{1 + K_p}$

Lag compensation improves $K_p$, $K_v$ and $K_a$ by $\dfrac{z_c}{p_c}$ IE: $K_{pc} \simeq K_{puc} \cdot \dfrac{z_c}{p_c}$

angle of constant damping line: $90 \cdot deg + \operatorname{atan}\left(\dfrac{a}{b}\right)$

# Bode Plots

Sample transfer function: $P(s) = K\cdot\frac{(s+z_1)\cdot(s+z_2)}{s\cdot(s+p_1)\cdot(s+p_2)^2}$

Replace all s's with blanks: $P(\_) = K\cdot\frac{(\_+z_1)\cdot(\_+z_2)}{\_\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+p_2)}$

Initial magnitude $= K\cdot\frac{z_1\cdot z_2}{(j\cdot\omega_{start})\cdot p_1\cdot(p_2)\cdot(p_2)}$

ω's in num --> + 20dB/dec each , j => 90º, - => +180º

ω's in den --> - 20dB/dec each , j => -90º, - => -180º

Cross out each pole or zero in turn & replace with jω $K\cdot\frac{(\_+z_1)\cdot(\_+z_2)}{(j\omega)\cdot(j\omega+X_1)\cdot(\_+p_2)\cdot(\_+p_2)}$

zeros turn up the slope --> + 20dB/decade
poles turn down the slope --> - 20dB/decade

zeros increase the phase angle --> + 90deg
poles decrease the phase angle --> - 90deg

Draw a smooth line through the bode plots to estimate the actual magnitude and phase.

Actual magnitude: -3dB at single poles, +3dB at single zeros; -6dB at double poles, +6dB at double zeros; etc.. Magnitude effects extend about 1 decade fore and aft.

Angle effects extend about 1.5 decade fore and aft. It is helpful to draw a line from 1 decade before to 1 decade after.

$$(s+a)^2+b^2 = s^2+2\cdot a\cdot s+a^2+b^2$$
$$= s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}$$

natural frequency $\omega_n = \sqrt{a^2+b^2}$

damping factor $\zeta = \frac{a}{\omega_n}$

max at approx $\omega_n$ , $\frac{1}{2\cdot\zeta}$

in dB: $20\cdot\log\left(\frac{1}{2\cdot\zeta}\right)$