## ECE 3510 Final given: Spring 16

1. (15 pts) When an electrical circuit is used as a representation if a mechanical system of translational motion, what do the following electrical quantities or parts represent in the mechanical system?
a) Current source =
d) Ground =
b) Branch current =
e) Resistor $=$
c) Nodal voltage =
f) Inductor =
g) Capacitor $=$

Also h) Is the capacitor always hooked up in a certain way? If yes, say what.
i) Name two things represented by transformers. You may include items that rotate. 1.
2.
2. (12 pts) For each of the time-domain signals shown, draw the poles of the signal's Laplace transform on the axes provided. All time scales are the same. The axes below all have the same scaling. Your answers should make sense relative to one another. Clearly indicate double poles if there are any.
a)

b)

3. (12 pts) Several transfer functions are shown below. Without doing
Y) definitely BIBO table anything more than looking at the transfer function, try to determine
N) definitely NOT BIBO table if it is BIBO stable. Answer Y, N, or C for each.
C) can't tell just by looking
a) $\frac{s \cdot(s+6)}{s^{5}+4 \cdot s^{4}+2 \cdot s^{3}+6 \cdot s^{2}+2 \cdot s+1}$
b) $\frac{s+1}{s^{5}+3 \cdot s^{4}-18 \cdot s^{3}+3 \cdot s^{2}+s+2}$
c) $\frac{\mathrm{s}-4}{\mathrm{~s}^{4}+2 \cdot \mathrm{~s}^{3}+1 \cdot \mathrm{~s}^{2}+\mathrm{s}+0.25}$
e) $\frac{10 \cdot s+1}{(s+2) \cdot\left(s^{2}+2 \cdot s+4\right) \cdot s}$
d) $\frac{s^{4}+10 \cdot s^{2}+s}{\left(s^{2}+2 \cdot s+5\right) \cdot\left(s^{2}+4 \cdot s+4\right)}$
f) $\frac{s+2}{5 \cdot s^{6}+13 \cdot s^{4}+8 \cdot s^{3}+s^{2}+s+2}$
$\qquad$
$\qquad$
4. (12 pts) a) Sketch the root-locus plot for the following open-loop transfer function: Use only the rules you were told to memorize, that is, you may estimate details like breakaway points and departure angles from complex poles. Show your work where needed (like calculation of the centroid). Number each axis.
$\mathbf{G}(\mathrm{s})=\frac{s^{2}+2 \cdot s+5}{(s-1) \cdot(s+6) \cdot\left(s^{2}+10 \cdot s+29\right)}$
b) Find the range of gain (k) for which the system is closed-loop stable. Assume $\mathrm{k}>0$.

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5. (12 pts) Sketch the Bode plot for the following transfer function. Make sure to label the graphs, and to give the slopes of the lines in the magnitude plot. Also draw the "smooth" lines.
a) $\quad P_{a}(s)=\frac{s \cdot(s+2000)}{(s+4) \cdot(s+100)}$

6. (9 pts) A Nyquist curve is shown at right (only the portion for $\omega>0$ is plotted).
a) Knowing that the closed-loop system is stable, must the open-loop system be stable?
b) How many unstable poles does the open loop system have?
c) What is (are) the gain margin(s)?

7. (16 pts) All parts of this problem refer to the system whose Nyquist curve is shown at right (only the portion for $\omega>0$ is plotted). Recall that the Nyquist curve represents the frequency response of the open-loop system, or $\mathrm{G}(\mathrm{j} \omega)$. If $\mathrm{G}(\mathrm{s})$ is the open-loop transfer function. The closed-loop transfer function is $\mathrm{G}(\mathrm{s}) /(1+\mathrm{G}(\mathrm{s})$ ). The open-loop system is stable. Show work for each part. P:=0
a) Is the closed-loop system stable?
b) How many unstable poles does the closed-loop system have?
c) Give the steady-state response $y_{\text {ss }}(t)$ of the open-loop system to an input $\mathrm{x}(\mathrm{t})=5 \cos (7 \mathrm{t})$.


The gain is reduced by a factor of two (multiplied by 0.5 ) for the remainder of this problem.
d) Is the closed-loop system stable now?
e) Estimate the phase margin of the closed-loop system.
f) Give the steady-state response $y s s(t)$ of the closed-loop system to an input $x(t)=8$

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1. (13 pts) Sketch the Bode plot for the following transfer function. Make sure to label the graphs, and to give the slopes of the lines in the magnitude plot. Also draw the "smooth" lines.
$\mathrm{P}_{\mathrm{a}}(\mathrm{s})=\frac{\left(\mathrm{s}^{2}+10 \cdot \mathrm{~s}+10000\right)}{(\mathrm{s}+2) \cdot(\mathrm{s}+30)}$

2. (14 pts) Given the magnitude Bode plot of a system, estimate of the transfer function of the system. Assume there are no negative signs in the transfer function (all poles and zeros are in the left-half plane). Show your work (how you made your estimate).

3. (12 pts) The open-loop Bode plots of a system are given at right.
a) Find the gain margin and phase margin of the closed-loop system. Show your work on the drawings.
b) Find the delay margin.

4. (18 pts) The unit-step response of a system is shown below as a function of time. Note: unit-step means $\mathrm{x}_{\mathrm{m}}:=1$

a) Find to Laplace transform of the unit-step response, $\mathbf{Y}(\mathrm{s})$. Show your work on the drawing.

Express $\mathbf{Y}(\mathrm{s})$ as precisely as you can, finding as many numbers as you can. If there is anything that you know must be a part of $\mathbf{Y}(\mathrm{s})$, but you cannot find as a number, express it as a letter constant (a or b or c etc.) Hint: Can you determine the time constant, ringing frequency and final value from the drawing?
b) Find system transfer function, $\mathbf{H}(\mathrm{s})$.

Express $\mathbf{H}(\mathrm{s})$ as precisely as you can, finding as many numbers as you can. If there is anything that you know must be a part of $\mathbf{H}(\mathrm{s})$, but you cannot find as a number, express it as a letter constant (a or b or c etc.)

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5. ( 35 pts ) a) Sketch the root locus plot of, $\mathrm{G}(\mathrm{s}):=\frac{100}{(\mathrm{~s}+25) \cdot(\mathrm{s}+40) \cdot(\mathrm{s}+70)}$

The gain is set at 452 , so that one of the closed-loop poles is at,

$$
\mathrm{s}:=-24.48+27.2 \cdot \mathrm{j}
$$


c) Add a LEAD compensator so that you will be able to place the closed-loop pole at the location found in b).

Add the new zero at -30 . Find the location of the new pole.

d) With the compensator in place and a closed-loop pole at the location desired in part b)
i) What is the gain?
ii) What is the $2 \%$ settling time? Use the second-order approximation.
iii) What is the steady-state error to a unit-step input?
e) Add another compensator: $C_{2}(s):=\frac{s+2}{s}$ and maintain the gain of part d)
i) What is this type of compensator called and what is its purpose?
ii) Calculate what you need to to show that this compensator achieved its purpose.
f) With both compensators in place, is there possibility for improvement (quicker settling time speed and/or lower ringing)? If yes, what would be the simplest thing to do? Justify your answer.

Do you want your grade and scores posted on the Internet?
otherwise, leave blank
The grades will be posted on line in pdf form in alphabetical order under the alias that you provide here. I will not post grades under your real name or an alias that looks like a real name or u-number. The pdf spreadsheet will show the homework, lab, and exam scores of everyone who answers here.

## Answers

d) Stationary reference of zero velocity

1. a) Force input
e) Friction or damping
c) Velocity
f) Spring
g) Mass
2. a)

3. 



6. a) NO, the open-loop system is not stable
b) 2
c) $8 / 11<$ gain $<3 / 2$
7. a) No, it is unstable
b) 2
c) $10 \cos \left(7 t-135^{\circ}\right)$.
d) yes
e) $45 \cdot \mathrm{deg}$
f) 6

$\begin{array}{ll}\text { a) } C & \text { b) } N \\ \text { c) } C & \text { d) } Y \\ \text { e) } N & \text { f) } N\end{array}$
h) Yes, one side is always hooked to ground
i) 2 of these: Levers Wheels Belts Gears Electric motors
4. a)
b) $k>34.8$

## Closed-Book



2. $\frac{10 \cdot s \cdot(s+500)}{\left(s^{2}+0.6 \cdot s+9\right)}$
3. a) $19 \cdot d B \quad 96 \cdot \mathrm{deg}$
b) $186 \cdot \mathrm{~ms}$
4. a) $\frac{12.566}{s^{2}+0.5 \cdot s+1.642}$
b) $\frac{12.566 \cdot \mathrm{~s}}{\mathrm{~s}^{2}+0.5 \cdot \mathrm{~s}+1.642}$
b) $-36+40 \cdot j$
c) $\mathbf{C}(\mathrm{s})=\frac{(\mathrm{s}+30)}{(\mathrm{s}+85)}$
d) i) 1369
ii) $0.111 \cdot \mathrm{sec}$
iii) $59.2 . \%$
e) i) PI, used to eliminate steady-state error
ii) $\mathrm{e}_{\text {step }}=\frac{1}{1+\mathrm{k} \cdot \infty}=0 . \%$
f) A quick sketch of the new root-locus shows that simply decreasing the gain would improve the system

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