## ECE 3510 Final given: Spring 22

1. (15pts) The op-amp in the circuit shown has a gain bandwidth product of 2 MHz , an open-loop DC gain of 100 dB , an input resistance of $100 \mathrm{M} \Omega$, and an output resistance of $50 \Omega$.
a) What is the feedback factor (B) and the gain of the circuit shown at right.

b) What is the 3 dB roll-off point of this circuit?
c) What is the input resistance of this circuit?
d) What is the output resistance of this circuit?
e) What does "compensation" mean when refering to op-amps, and why are op-amps compensated?
2. (10pts) Find the input impedance $\left(\mathbf{Z}_{\mathbf{i n}}(\mathbf{s})\right)$ and output impedance $\left(\mathbf{Z}_{\mathbf{o u t}}(\mathbf{s})\right)$ of the circuits below. You don't have to simplify the results in any way, not even combining terms with a common denominator.

$\qquad$
3. (20 pts) For each of the following discrete-time signals, draw the poles on the z-plane shown. A unit circle is shown on each z-plane as a dotted line.

Fin S22
a)

b)


c)

。

d)


e)


4. (30 pts) Sketch the Bode plot for the following transfer function. Make sure to label the graphs as necessary to show the magnitudes and slopes. Also draw the "smooth" lines.

Fin S22
a) $\mathbf{P}(\mathrm{s}):=\frac{10^{5} \cdot \mathrm{~s} \cdot(\mathrm{~s}+100)}{(\mathrm{s}+2) \cdot(\mathrm{s}+1000)^{2}}$

At each corner, show (in dB ) the difference between the straight lines and the smooth line.


b) $\mathbf{P}(\mathrm{s}):=\frac{10^{6} \cdot\left(\mathrm{~s}^{2}+2 \cdot \mathrm{~s}+25\right) \cdot(\mathrm{s}+300)}{\mathrm{s}^{2} \cdot\left(\mathrm{~s}^{2}+1000 \cdot \mathrm{~s}+10^{8}\right)}$


5. (35 pts) Consider this transfer function.
a) Sketch the root-locus plot

$$
\mathbf{G}(\mathrm{s}):=\frac{\mathrm{s}+8}{\mathrm{~s} \cdot(\mathrm{~s}+5) \cdot(\mathrm{s}+20)}
$$


b) You wish to add a compensator to get closed-loop damping factor of 0.7071 if the ringing is $12 \mathrm{rad} / \mathrm{sec}$. (using the second-order approximation)


Note: If you can't calculate the zero location or doubt your calculation, assume it is at -35 for the rest of this problem. For the remaining problem, the compensator in place and a closed-loop pole at the location desired in part b)
c) What is the gain?
d) What is the steady-state error to a unit-step input?
e) With this compensator in place, is there possibility for improvement (better speed and/or lower ringing)? If yes, what would be the simplest thing to do? Justify your answer.
6. ( 14 pts ) Discrete-time signals are shown below. Find $\mathrm{x}(\mathrm{k})$ and the z -transforms $\mathbf{X}(\mathrm{z})$ of these signals. Find the $z$-transform in closed form (not a series). Simplify to standard form.
$\mathrm{x}(\mathrm{k})$

$\qquad$ / 14
7. (8 pts) Use partial fraction expansion to find $\mathrm{x}(\mathrm{k})$ for the following z -transform:
$\mathbf{X}(z)=\frac{6}{(z-0.5) \cdot(z+1)}$
8. (25 pts) a) Use partial fraction expansion to find $x(k)$ for the following z-transform:

$$
\mathbf{X}(z)=\frac{z \cdot(z+0.3)}{(z-1) \cdot\left(z^{2}-0.8 \cdot z+0.97\right)}
$$

b) Is the signal bounded?

Does it converge?
If yes, to what value?
9. (23 pts) a) Draw the block diagram of a simple direct implementation of the difference equation.

$$
\mathrm{y}(\mathrm{k})=3 \cdot \mathrm{x}(\mathrm{k})-2 \cdot \mathrm{x}(\mathrm{k}-2)+\mathrm{x}(\mathrm{k}-3)+\frac{1}{2} \cdot \mathrm{y}(\mathrm{k}-1)-\frac{1}{4} \cdot \mathrm{y}(\mathrm{k}-3)
$$

b) Find the $\mathbf{H}(z)$ corresponding to the difference equation above. Show your work. Simplify your expression for $\mathbf{H}(z)$ so that the denominator is a simple polynomial or a multiple of simple polynomials.
c) List the poles of $\mathbf{H}(\mathrm{z})$. Indicate multiple poles if there are any. Note: If your calculator can't find the roots needed, go see Arn with the equation that you need solved.
d) Is this system BIBO stable? Yes No How do you know?
e) Is this an FIR system? Yes No
 If not, which terms in the difference equation are to blame?

Answers

1. a) 16
c) $100 \cdot \mathrm{M} \Omega$
b) $125 \cdot \mathrm{kHz}$ d) $0.008 \cdot \Omega$
e) Limit the bandwidth with a simple single-pole low-pass
filter. Done so that the op-amp circuit will still have a
decent phase margin, even with a feedback factor of 1 .
2. a)
 4. a)


$\omega(\mathrm{rad} / \mathrm{sec})$
b)

Fin S22
2. $\mathrm{R}_{2}+$
$\frac{1}{\mathrm{R}_{1}}+\mathrm{C} \cdot \mathrm{s}+\frac{1}{\mathrm{~L} \cdot \mathrm{~s}}$



b) $\mathbf{C}(\mathrm{s}):=(\mathrm{s}+40.091)$
c) 8.8
d) $0 \%$
e) A quick sketch of the new root-locus shows that simply increasing the gain would improve the system
6. $\frac{z \cdot(10 \cdot z-8)}{(z-1) \cdot(z-0.5)}$
7. $\left[-12 \cdot \delta(\mathrm{k})+8 \cdot(0.5)^{\mathrm{k}}+4 \cdot(-1)^{\mathrm{k}}\right] \cdot \mathrm{u}(\mathrm{k})$
9. a)

b) $\frac{3 \cdot z^{3}-2 \cdot z+1}{z^{3}-\frac{1}{2} \cdot z^{2}+\frac{1}{4}}$
c) $-0.5 \quad 0.5+0.5 \cdot \mathrm{j} \quad 0.5-0.5 \cdot \mathrm{j}$
d) Yes All poles are inside the unit circle
e) No $\quad \frac{1}{2} \cdot \mathrm{y}(\mathrm{k}-1) \quad$ and $\quad-\frac{1}{4} \cdot \mathrm{y}(\mathrm{k}-3)$

