ECE 3510 Finish Ch 2

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1/25/09
1/19/16
1/21/23

Non-strictly-proper transforms

section 2.2.5, p.16 in Bodson text

What if the order of the numerator is equal to or even greater than the order of the denominator? $m \geq n$?

can't work ! no s^2 term in numerator !

Example: $F(s) = \dfrac{2\cdot s^2 + 100}{s^2 + 8\cdot s + 41}$ $\quad m := 2 \quad n := 2$ $\quad \overset{?}{=} \dfrac{A}{s+4+5\cdot j} + \dfrac{B}{s+4-5\cdot j} = \dfrac{A\cdot(s+4+5\cdot j) + B\cdot(s+4-5\cdot j)}{s^2+8\cdot s+41}$

First divide, before partial fraction expansion

$$\begin{array}{r|l} & \quad 2 \\ s^2 + 8\cdot s + 41 & \; 2\cdot s^2 + 0\cdot s + 100 \\ & -2\cdot\left(s^2 + 8\cdot s + 41\right) \\ \hline \text{"remainder"} & \; 0\cdot s^2 - 16\cdot s + 18 \end{array}$$

$$F(s) = \frac{2\cdot s^2 + 100}{s^2 + 8\cdot s + 41} = 2 + \frac{-16\cdot s + 18}{s^2 + 8\cdot s + 41}$$

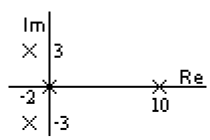
$$f(t) = 2\cdot\delta(t) + \left(\frac{82}{5}\cdot e^{-4\cdot t}\cdot \sin(5\cdot t) - 16\cdot e^{-4\cdot t}\cdot\cos(5\cdot t)\right)\cdot u(t)$$

Delta functions are not very common in real life.
Non-strictly-proper transforms are just as common.

Properties of Signals

Can you tell what f(t) must be just by looking at F(s)? YES, somewhat...
SEE last page of Lect 2 & 3 notes

$$\frac{s+5}{s\cdot\left(s^2+4\cdot s+13\right)\cdot(s-10)}$$



s means a pole at origin, so output has some DC of unknown $\pm$ value, and may converge to a non-zero (DC) value

$\left(s^2+4\cdot s+13\right) = (s+2+3\cdot j)\cdot(s+2-3\cdot j)$ Decaying cosine wave of unknown phase or amplitude

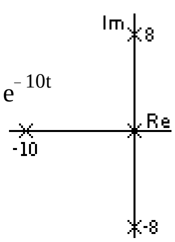
e^{-2t} decay rate, time constant = $1/2$ sec, frequency = 3 rad/sec or $3/2\pi$ Hz

$(s-10)$ means signal is unbounded and (of course) doesn't converge

$$\frac{s+5}{s\cdot\left(s^2+64\right)\cdot(s+10)}$$

DC

e^{-10t}



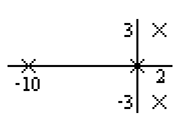
$\left(s^2+64\right) = (s+8\cdot j)\cdot(s-8\cdot j)$

Cosine wave of unknown phase or amplitude, frequency = 8 rad/sec or $4/\pi$ Hz
means signal is bounded but doesn't converge

$s+10$ means an e^{-10t} decaying exponential is part of the signal

$$\frac{s+5}{s\cdot\left(s^2-4\cdot s+13\right)\cdot(s+10)}$$

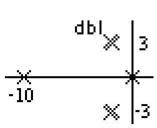
$s^2 - 4\cdot s + 13 =$
$(s-2+3\cdot j)\cdot(s-2-3\cdot j)$



DC and e^{-10t}

Cosine wave of unknown phase and exponentially increasing amplitude

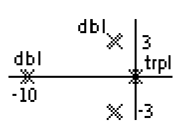
$$\frac{s+5}{s\cdot\left(s^2+4\cdot s+13\right)^2\cdot(s+10)} = \frac{A}{s} + \frac{B\cdot(s+2)+C\cdot 3}{\left(s^2+4\cdot s+13\right)} + \frac{D\cdot\left[(s+2)^2-3^2\right]+E\cdot(6\cdot(s+2))}{\left(s^2+4\cdot s+13\right)^2} + \frac{F}{(s+10)}$$



$$\left[A + B\cdot e^{2\cdot t}\cdot\cos(3\cdot t) + C\cdot\left(e^{2\cdot t}\cdot\sin(3\cdot t)\right) + D\cdot t\cdot e^{2\cdot t}\cdot\cos(3\cdot t) + E\cdot t\cdot e^{2\cdot t}\cdot\sin(3\cdot t)\right]\cdot u(t)$$

A and F cannot be zero, neither can both D and E ($D^2 + E^2 > 0$)

$$\frac{s+5}{s^3\cdot\left(s^2+4\cdot s+13\right)^2\cdot(s+10)^2}$$



s^3 term results in a rising t^2 in time domain
as well as possible DC and/or ramp