

1. Find the inverse Laplace transform of each of the following functions: Use the mixed method and the tables.

a)
$$F(s) = \frac{3s + 6}{(s^2 + 1)(s^2 + 4)}$$

b)
$$F(s) = \frac{1}{(s + 2)(s + 1)^2}$$

c) $F(s) = \frac{2 \cdot s}{s^2 + 2 \cdot s + \frac{5}{4}}$

d) $F(s) = \frac{8 \cdot s + 4}{s^2 \cdot (s + 1)^2}$

$$\text{e) } F(s) = \frac{\frac{1}{2} \cdot s^3 + s^2 + s + \frac{5}{2}}{s^2 \cdot (s^2 + 2 \cdot s + 5)}$$

2. $F(s) = \frac{s - 1}{s^3 \cdot (s^2 + 2s + 5)^2}$

Show the form of $f(t)$ without actually finding it. Indicate which of the coefficients may not be 0

3. Problem 2.3 a) - g) in textbook (p.21) As part of your work to reach a solution, draw the pole diagram for each.

a) $\frac{10}{(s+1)^{10}}$		b) $\frac{(s-1)}{s \cdot (s+2)}$		c) $\frac{1}{s^2 \cdot (s+2)}$		d) $\frac{5}{s \cdot (s+1)^2}$	
e) $\frac{3}{s \cdot (s^2 + 4)}$		f) $\frac{3}{s \cdot (s^2 + 4)^2}$		g) $\frac{2 \cdot (s-1)}{(s^2 + 2s + 1) \cdot (s+3)}$			

Bounded Converges $f(\infty)$

- a)
- b)
- c)
- d)
- e)
- f)
- g)

Answers (time functions below valid for $t \geq 0$ only)

1. a) $(\cos(t) + 2 \cdot \sin(t) - \cos(2t) - \sin(2t)) \cdot u(t)$ b) $(e^{-2t} + t \cdot e^{-t} - e^{-t}) \cdot u(t)$
 c) $\left(2 \cdot e^{-t} \cdot \cos\left(\frac{1}{2} \cdot t\right) - 4 \cdot e^{-t} \cdot \sin\left(\frac{1}{2} \cdot t\right)\right) \cdot u(t)$ d) $(4 \cdot t - 4 \cdot t \cdot e^{-t}) \cdot u(t)$ e) $\left(\frac{1}{2} \cdot t + \frac{1}{2} \cdot e^{-t} \cdot \cos(2t)\right) \cdot u(t)$
2. $(A + B \cdot t + C \cdot t^2 + D \cdot e^{at} \cdot \cos(bt) + E \cdot e^{at} \cdot \sin(bt) + F \cdot t \cdot e^{at} \cdot \cos(bt) + G \cdot t \cdot e^{at} \cdot \sin(bt)) \cdot u(t)$

C may not be 0 & Either F or G may be 0, but **NOT BOTH**

Alternate solution: $\left(A + B \cdot t + C \cdot t^2 + \sqrt{D^2 + E^2} \cdot e^{at} \cdot \cos(b \cdot t + \theta) + \sqrt{F^2 + G^2} \cdot t \cdot e^{at} \cdot \cos(b \cdot t + \phi)\right) \cdot u(t)$

3.	<u>Bounded</u>	<u>Converges</u>	$f(\infty)$	<u>Bounded</u>	<u>Converges</u>	$f(\infty)$	Can't be 0:	C	&	$\sqrt{F^2 + G^2}$
a)	yes	yes	0	d)	yes	5				
b)	yes	yes	$-\frac{1}{2}$	e)	yes	no				
c)	no			f)	no					
				g)	yes	yes	0			