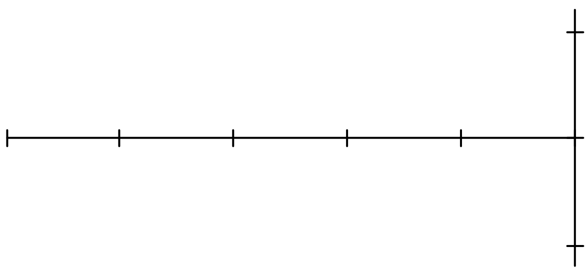
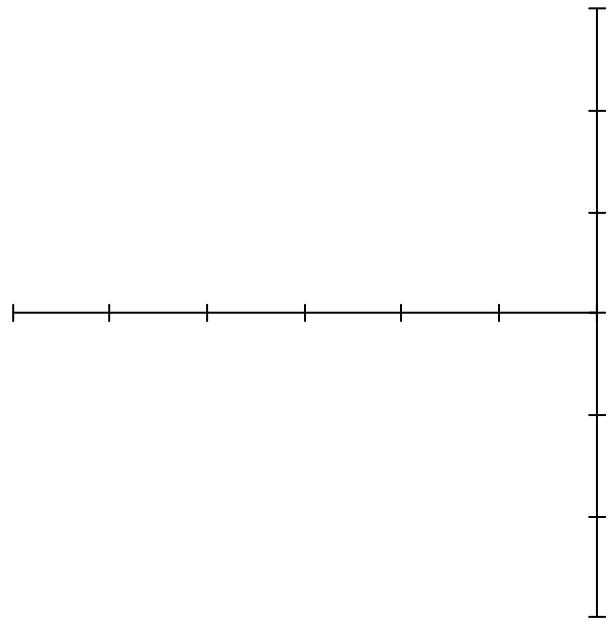
Name ____________________________ ECE 3510 homework RL3 Due: Wed, 2/28/24 b

1. Problem 4.5 (p.109) in the text.
   Sketch (by hand) the root-locus plot for the following open-loop transfer functions:
   Apply only the main rules (Section 4.6.2 in text or the first page of my notes)

a) $G(s) = \dfrac{s\cdot(s+1)}{(s+2)^2\cdot(s+3)}$

b) $G(s) = \dfrac{(s+3)}{s\cdot(s+9)^3}$





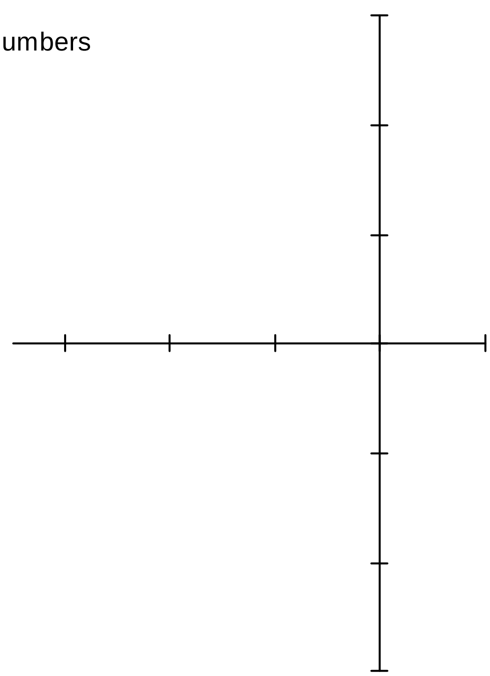
c) $G(s) = \dfrac{(s+a)}{(s+b)\cdot\left(s^2-2\cdot s+2\right)}$

$a > 0$ $b > 0$ $k > 0$

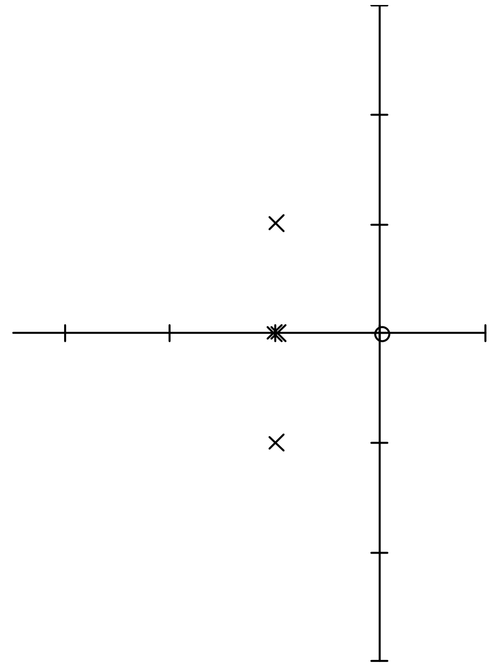
a, b, & k are all positive, real numbers

$s^2 - 2\cdot s + 2 = (s - 1 - j)\cdot(s - 1 + j)$

Also give condition(s) that a and b must satisfy for the closed-loop system to be stable for sufficiently high gain (k) (note that you do not need to apply the Routh-Hurwitz criterion, nor provide the range of k for which the system is closed-loop stable).



2. Problem 4.12 in the text.
   Sketch the root-locus for the open-loop poles shown at right, using only the main rules. There is a zero at $s = 0$, two poles at $s = -1$ and two poles at $s = -1 \pm j$.



The following review questions and problems come from the Nise 3rd Ed., starting on page 471,
Or 4th Ed., starting p 473. If you are using the 4th Ed., <u>clearly</u> state that on your homework.

3. Nise, Ch.8, review question 3, rephrased here: If $G(s_1) = 5\,\underline{/\,180^{\circ}}$, is the point s_1 on the root locus?
   If yes, what gain factor would place a closed-loop pole at s_1?

4. Nise, Ch.8, review questions 4, 6, 7, 8, 9, 10.

   4. Do the zeros of a system change with a change in gain?

   6. What are two ways to find where the root locus crosses the imaginary axis?

      (1)

      (2)

   7. How can you tell from the root locus if a system is unstable?

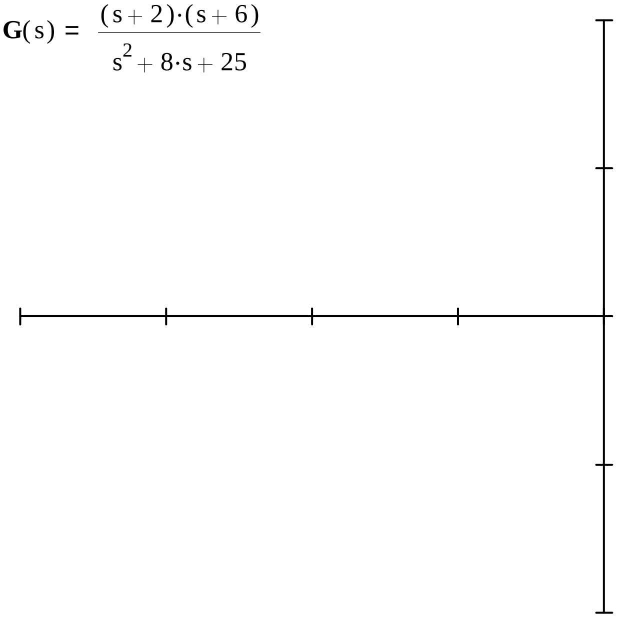
   8. How can you tell from the root locus if the settling time does not change over a region of gain?

   9. How can you tell from the root locus that the natural frequency does not change over a region of gain?

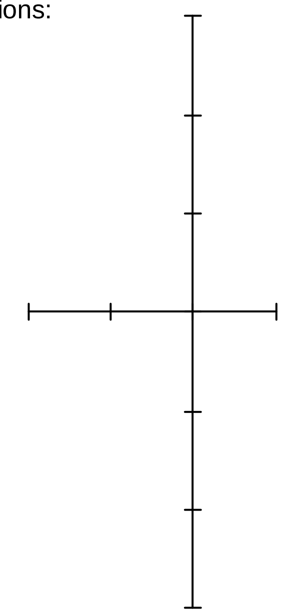
   10. How would you determine whether or not the root locus plot crossed the real axis?

5. Nise, Ch.8, problem 3   Sketch the root locus for the following open-loop transfer functions:

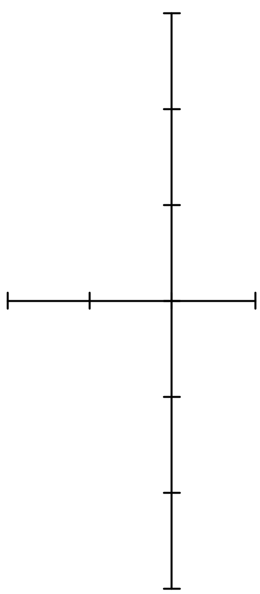
a) $\mathbf{G}(s) = \dfrac{(s+2)\cdot(s+6)}{s^2+8\cdot s+25}$



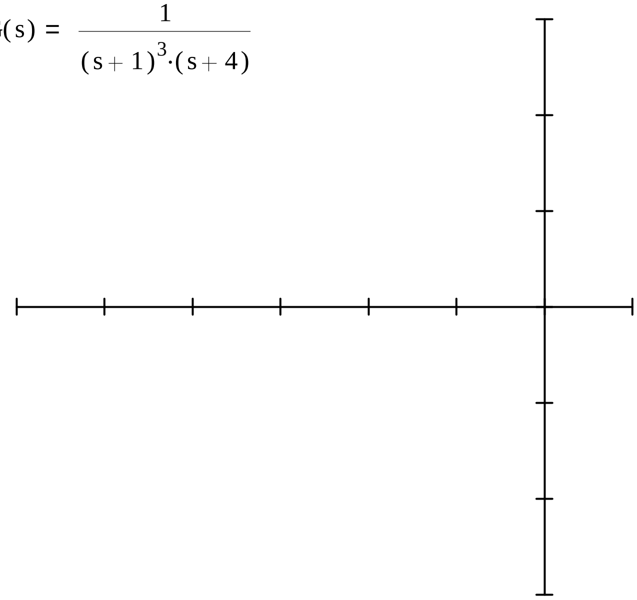
b) $\mathbf{G}(s) = \dfrac{s^2+2^2}{s^2+1^2}$



c) $\mathbf{G}(s) = \dfrac{s^2+1^2}{s}$



d) $\mathbf{G}(s) = \dfrac{1}{(s+1)^3\cdot(s+4)}$



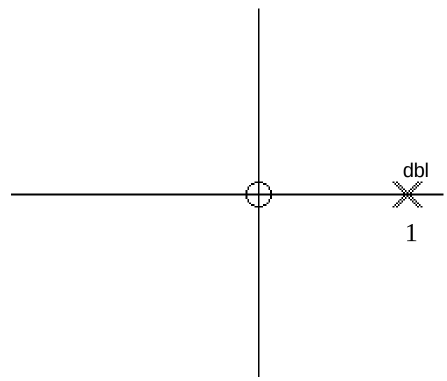
6. a) NISE Ch.8 problem 3a   Find the break-in point for Nise, problem 3a, above. Note, the math here may drive you nuts, but you may simply test and prove that a point you guess is correct.

6. b) Find the break-away point for Nise, problem 3d, above.

NISE Ch.8 problem 3d $\mathbf{G}(s) = \dfrac{1}{(s+1)^3 \cdot (s+4)}$

7. Problem 4.11 in the Bodson text. (Hint: do part c before b)

a) Sketch the root-locus for the open-loop poles shown at right. There is one zero at $s = 0$ and two poles at $s = 1$.

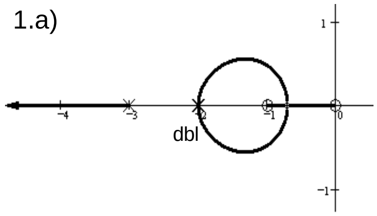


b) Give the range of gain k ($k > 0$) for which the system is closed-loop stable, and give the locations of the $j\omega$ axis crossings.

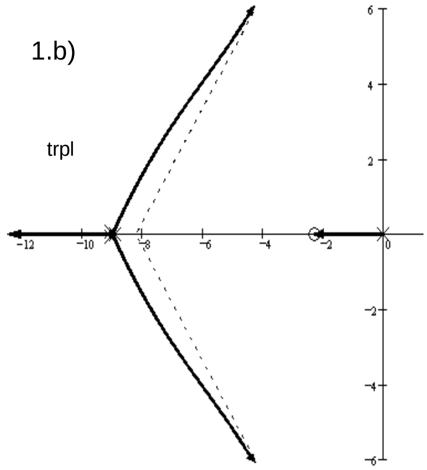
c) Give the location of the break-away point (arrival) on the real axis.

Answers

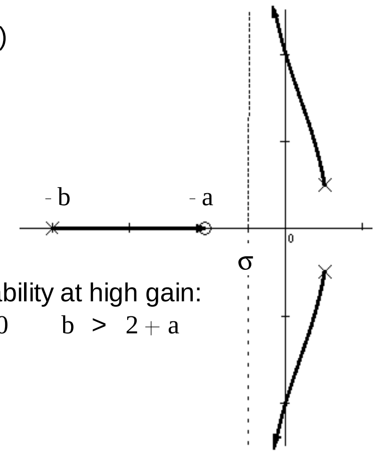
1.a)



1.b)

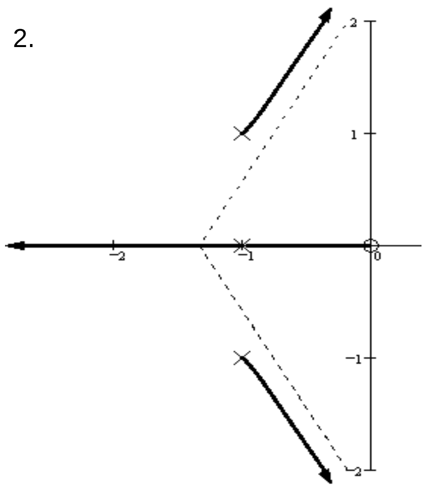


1.c)

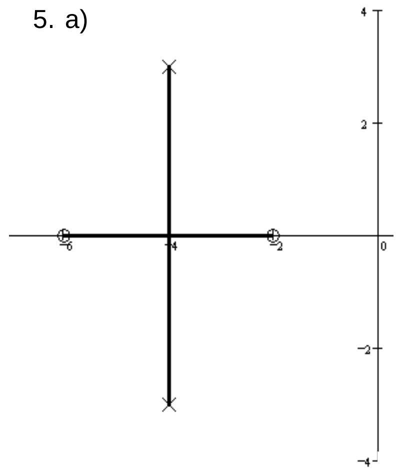


Stability at high gain:
a>0 b > 2 + a

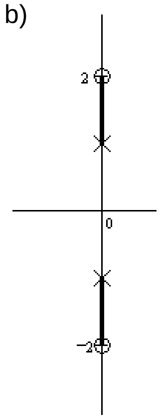
2.



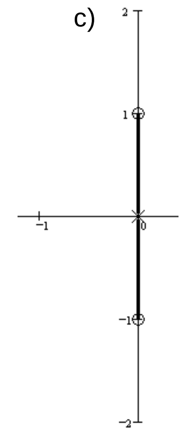
5. a)



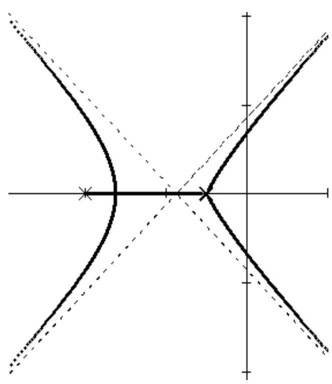
b)



c)



5. d)

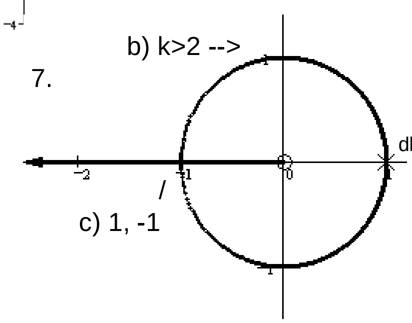


6. a) - 4

b) - 3.25

7.

b) k>2 -->



c) 1, -1

ANSWERS TO REVIEW QUESTIONS
1. The plot of a system's closed-loop poles as a function of gain
2. (1) Finding the closed-loop transfer function, substituting a range of gains into the denominator, and factoring the denominator for each value of gain.
   (2) Search on the s-plane for points that yield 180 degrees when using the open-loop poles and zeros.
3. Yes, K = 1/5
4. No
5. At the zeros of G(s) and the poles of H(s)
6. (1) Apply Routh-Hurwitz to the closed-loop transfer function's denominator. (2) Search along the imaginary axis for $\angle G(s) = \pm 180^o$. (3) Use a computer with something like Matlab SISO tool to find the crossover point(s). (only need 2 ans)
7. If any branch of the root locus is in the rhp, the system may be unstable.
   If the gain places one of the closed-loop poles on that part of the branch, it will be unstable.
8. If the branch of the root locus is vertical, the settling time remains constant for that range of gain on the vertical section.
9. The natural frequency is the distance of a pole from the origin. If a region of the root locus is circular and the center of the circle is at the origin, then the natural frequency would not change over that region of gain.
10. Determine if there are any break-in or breakaway points
11. (1) Poles must be at least five times further from the imaginary axis than the dominant second order pair,
    (2) Zeros must be nearly canceled by higher order poles.
12. Number of branches, symmetry, starting and ending points
13. The zeros of the open loop system help determine the root locus. The root locus ends at the zeros.
    Thus, the zeros are the closed-loop poles for high gain.