These problems should be done using MATLAB or some other program that creates root-locus plots. Refer to a help sheet attaced to this homework to get started. You will need to compare shapes of root-locus (RL) plots and speeds of step responses. Unless you manually set your plot limits, Matlab will automatically scale on your plots. This can make it very hard to compare them or to determine the "45°" line.

1. a) Homework RL2 problem 1c. $\mathbf{G}(s) := \dfrac{1}{s\cdot(s+2)\cdot(s+4)}$ Experiment with moving the pole at -4.

   i) Describe what happens when you move it to the left.
   ii) Describe what happens when you move it to the right, especially when you move it right of -2.
       Put the pole back at -4.

   b) Describe what happens when you add a fourth pole left of the jω axis. Try different positions. Erase the added pole.

   c) Describe what happens when you add a zero left of the jω axis. Try different positions.

For the remaining problems:
   Print one or more plots for **each** problem. Each plot should be labeled clearly on the same page as the plot. If you added poles and/or zeros (a compensator) make sure it is clear which one(s) were added or what your $\mathbf{C}(s)$ is.   Plot of the root locus should always show the point(s) on the root locus you determined to be the "best" position for the CL pole(s) with squares (like Matlab does). Indicate the gain (k) used.
   "Best" = Fastest step response to get to and stay within 4% of the desired final response (usually 1 for step response). Limit overshoot to about 4%. To determine speed information, refer to the step response curve. Prints of those curves are nice, but not required. Be careful, if Matlab changes the scales on your plots it can be very hard to compare them.

2. a) Homework RL2 problem 1c (see above for $\mathbf{G}(s)$). Add a compensator to your system. This compensator will add one pole and one zero to the open-loop (OL) transfer function. The new pole must lie somewhere between -10 and +2, you choose where. Same goes for the zero. Choose the locations of your added OL pole and OL zero and gain (k) to get the "best" response.

   b) Homework RL2 problem 1d. $\mathbf{G}(s) := \dfrac{s+7}{s\cdot(s+2)\cdot(s+4)}$ Just find the "best" gain and print an RL plot.

   c) Homework RL2 problem 1d. Repeat part a) above for this system, only this time your added pole and zero are limited to -16 to +2.

3. a) Homework RL3 problem 1a $\mathbf{G}(s) = \dfrac{s\cdot(s+1)}{(s+2)^2\cdot(s+3)}$ Just print an RL plot. Note that the step response goes to zero, why?

   b) Homework RL3 problem 1b. $\mathbf{G}(s) = \dfrac{(s+3)}{s\cdot(s+9)^3}$ Just find the "best" gain and print an RL plot.

   c) Homework RL3 problem 1b. Add a compensator. Your compensator may have up to 2 poles (0, 1, or 2) and they may be complex. Same for the zeros. All must lie between -20 and +2 and   -12j and +12j. Try to choose the best possible poles and/or zeros, find the best gain and plot like you did for 1b. Don't spend too much time here, quickly decide between poles and zero, add 1 or 2, and tweak your gain. (2 zeros and ∞ gain looks good but isn't really practical.)

4. Homework RL4 $\mathbf{G}(s) = \dfrac{s+1}{s^2 - 2\cdot s + 2}$ Confirm the departure angles (With a small gain, place a CL pole very
   problem 1.   close to $1 + 1j$ and figure out the angle from its location.). Find the true Im-axis crossing point and the gain at that point. Find the break in point.

5. a) Enter in the plant $\mathbf{G}(s)$ of the crude servo: $\mathbf{G}(s) = \dfrac{1643}{s\cdot(s+16.64)\cdot(s+53.78)}$
   Add a PI compensator (a pole at 0 and a zero at -0.1).
   Find the gain for a 0.707 damping factor. Record the time it takes the step response to reach 1 the first time. You don't need any prints or plots.

   b) Add a zero at -16.64 and repeat, no prints or plot..

   c) Adjust the zero location further to the right while pushing the gain and watching the step response to find an even better combination. Again record the time it takes the step response to reach 1 the first time and save.
      Print the Root locus **and** the step response for this case.

   d) You will be using this device to build widgets. The time per widget is 10 times the time you recorded in each case. Your profit per widget is 1 cent. Your fixed costs per 8-hour day is $250. Describe the results to your company of each of the 3 compensators above.

6. Create the most interesting root locus plot that you can with no more than 10 poles and 10 zeros. Have some fun with this. You can go over 10 poles and zeros if you want, but the fewer the better to make the figure you want.

Answers

1. a) i) If the pole at -4 is moved further to the left:
   Curved part of RL straightens out because the centroid moves to the left.
   The gain required to reach any given damping factor goes up.
   The breakaway point moves closed to -1
   ii) If the pole at -4 is moved to the right the opposite effects occur, plus, if moved right of zero at -2 effects get worse, fast.
   iii) Adding a pole makes matters worse. It makes the curves bend down and makes a new curve bend off to the left.
   iv) Adding a zero makes matters better. Especially if placed on top of the pole at -2. Further to the right of that and it tends to make a dip in the step response. If that can be tolerated, then increased speeds are possible with increased gain.

2. a)
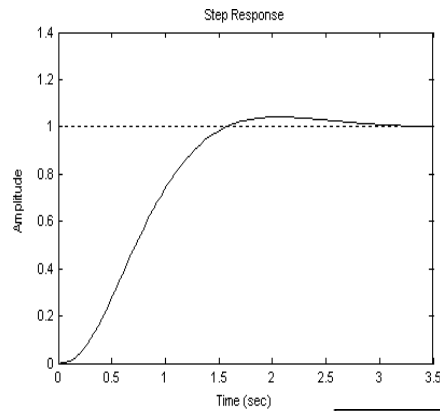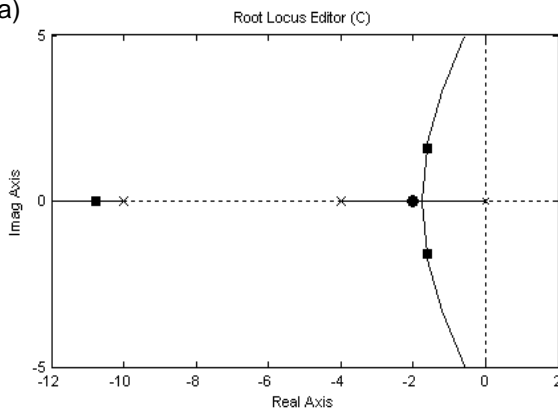
$$C(s) := \frac{s+2}{s+10}$$

Gain is about 56

Your answer may be different, especially, your zero may be to the right of -2.

b) See answers for RL2, gain is about 0.92   c) you're on your own

3. a) See answers for RL3   System has a zero at the origin.

   b) See answers for RL3

   Gain of 346 puts the CL poles on the 45º line, BUT, because of the zero at about -1.3, a gain of 1000 is a almost twice as fast.

   A gain of 800 with a feedforward gain of 1.04 will make it 4 times faster, but will result in a long-term +4% error in the output. If only we could limit the feedforward gain to the first part of the response. Ahhh... maybe that explains the weird feedback / feedforward scheme in lab 5b.

3b step response w/gain = 1000

3b step response w/gain = 800 and 1.04 feedforward

c)

gain = 253
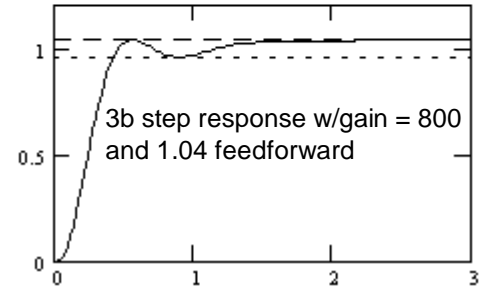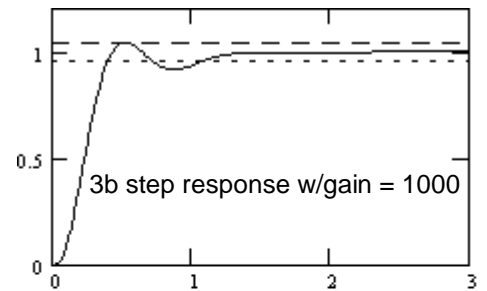
$$C(s) := \frac{s+9}{1}$$

gain = 253

$$C(s) := \frac{s+9}{1}$$ Your answer may be different

With gain = 253, overshoot is too high, gain = 215 meets requirements.
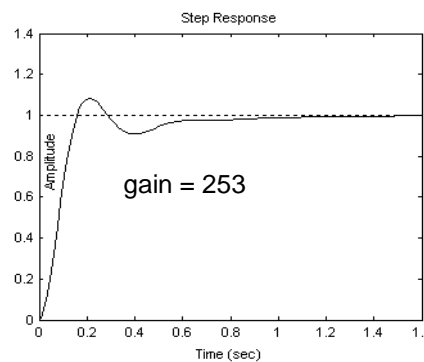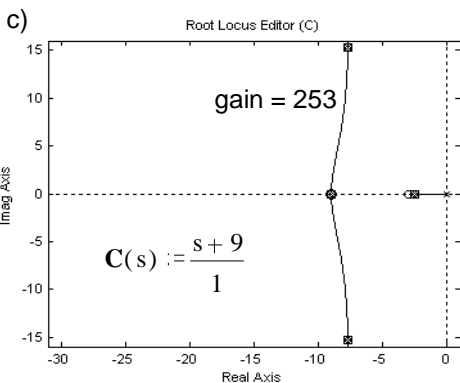
4. My close point: $s_1 := .945 + 1.1 \cdot j$ $\arg\left[s_1 - (1+j\ )\right] = 118.811 \cdot deg$ close to 117 deg

   Place CL pole at crossing, gain = 2
   Move CL pole to break in point, location shows at -3.23, at a gain of 8.47.

5.d) a. You go broke at $168/day.   b. You make $101/day.
   c. You make $161/day. Your answers may be quite different