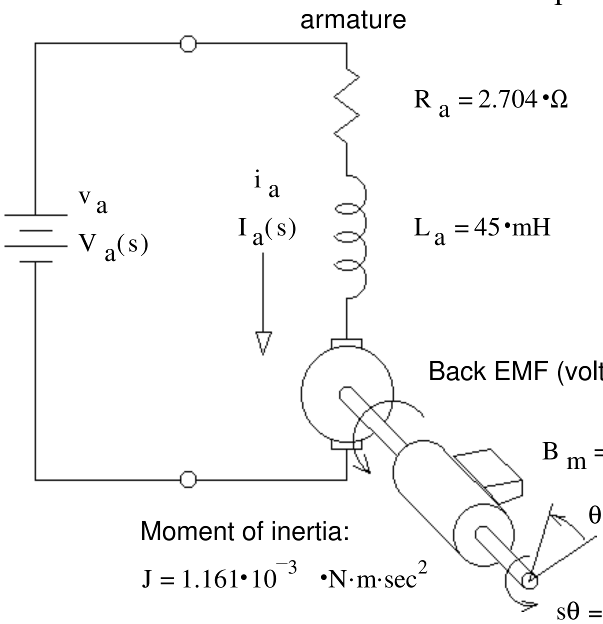
ECE 3510 Appendix: DC permanent-magnet motor & gears

A.Stolp
12/3/04
rev 3/23/06

For motor and gears taken together

Torque constant $K_T = 0.12 \cdot \frac{N \cdot m}{A}$ $K_V := K_T$ Voltage constant $K_V = 0.12 \cdot \frac{V}{\left(\frac{rad}{sec}\right)}$



$R_a = 2.704 \cdot \Omega$ Armature resistance

$L_a = 45 \cdot mH$ Armature inductance

Back EMF (voltage generated due to spinning) = $K_V \cdot s \cdot \theta(s)$

$B_m = 0.012 \cdot N \cdot m \cdot sec$ Viscous damping factor

θ = angle in radians

Moment of inertia:
$J = 1.161 \cdot 10^{-3} \cdot N \cdot m \cdot sec^2$

$s\theta$ = angular speed in radians/sec

$s^2\theta$ = angular acceleration in radians/sec^2

Torque: $$T(s) = J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s)) = K_T \cdot I_a(s) = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$$

$$J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s)) = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$$

$$\left[J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s))\right] \cdot \left(R_a + L_a \cdot s\right) = K_T \cdot \left(V_a(s) - K_V \cdot s \cdot \theta(s)\right)$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a = K_T \cdot V_a(s) - K_T \cdot K_V \cdot s \cdot \theta(s)$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a + K_T \cdot K_V \cdot s \cdot \theta(s) = K_T \cdot V_a(s)$$

$$\left(J \cdot s^2 \cdot R_a + J \cdot s^3 \cdot L_a + B_m \cdot s \cdot R_a + B_m \cdot s^2 \cdot L_a + K_T \cdot K_V \cdot s\right) \cdot \theta(s) = K_T \cdot V_a(s)$$

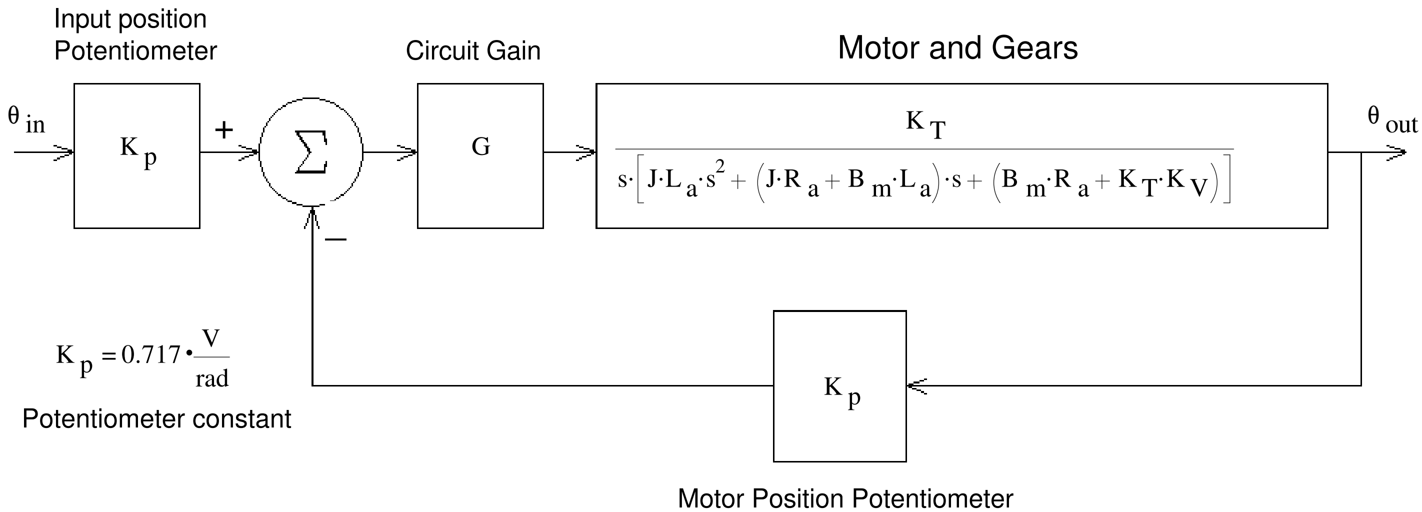
$$\left[J \cdot L_a \cdot s^3 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s^2 + \left(B_m \cdot R_a + K_T \cdot K_V\right) \cdot s\right] \cdot \theta(s) = K_T \cdot V_a(s)$$

Motor transfer function: $$\frac{\theta(s)}{V_a(s)} = \frac{K_T}{J \cdot L_a \cdot s^3 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s^2 + \left(B_m \cdot R_a + K_T \cdot K_V\right) \cdot s}$$

$$= \frac{K_T}{s \cdot \left[J \cdot L_a \cdot s^2 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s + \left(B_m \cdot R_a + K_T \cdot K_V\right)\right]}$$

ECE 3510 DC permanent-magnet motor, p2

The system block diagram



Overall System transfer function:

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = K_p \cdot \frac{G \cdot \dfrac{K_T}{s\cdot\left[J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)\right]}}{1 + K_p\cdot G\cdot\left[\dfrac{K_T}{s\cdot\left[J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)\right]}\right]}$$

$$\text{x}\quad \frac{s\cdot\left[J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)\right]}{s\cdot\left[J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)\right]}$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G\cdot K_T\cdot K_p}{s\cdot\left[J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)\right] + K_p\cdot G\cdot K_T}$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{k\cdot K_T\cdot K_p}{J\cdot L_a\cdot s^3 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s^2 + \left(B_m\cdot R_a + K_T\cdot K_V\right)\cdot s + K_p\cdot G\cdot K_T}$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{\dfrac{k\cdot K_T\cdot K_p}{J\cdot L_a}}{s^3 + \left(\dfrac{J\cdot R_a + B_m\cdot L_a}{J\cdot L_a}\right)\cdot s^2 + \left(\dfrac{B_m\cdot R_a + K_T\cdot K_V}{J\cdot L_a}\right)\cdot s + \dfrac{K_p\cdot G\cdot K_T}{J\cdot L_a}}$$

The characteristic equation: $0 = s^3 + p\cdot s^2 + q\cdot s + r$

Where: $p := \dfrac{J\cdot R_a + B_m\cdot L_a}{J\cdot L_a}$ $\quad q := \dfrac{B_m\cdot R_a + K_T\cdot K_V}{J\cdot L_a}$ $\quad r := \dfrac{K_p\cdot G\cdot K_T}{J\cdot L_a}$

ECE 3510 DC permanent-magnet motor, p3

for Gain = $G := 1.7$

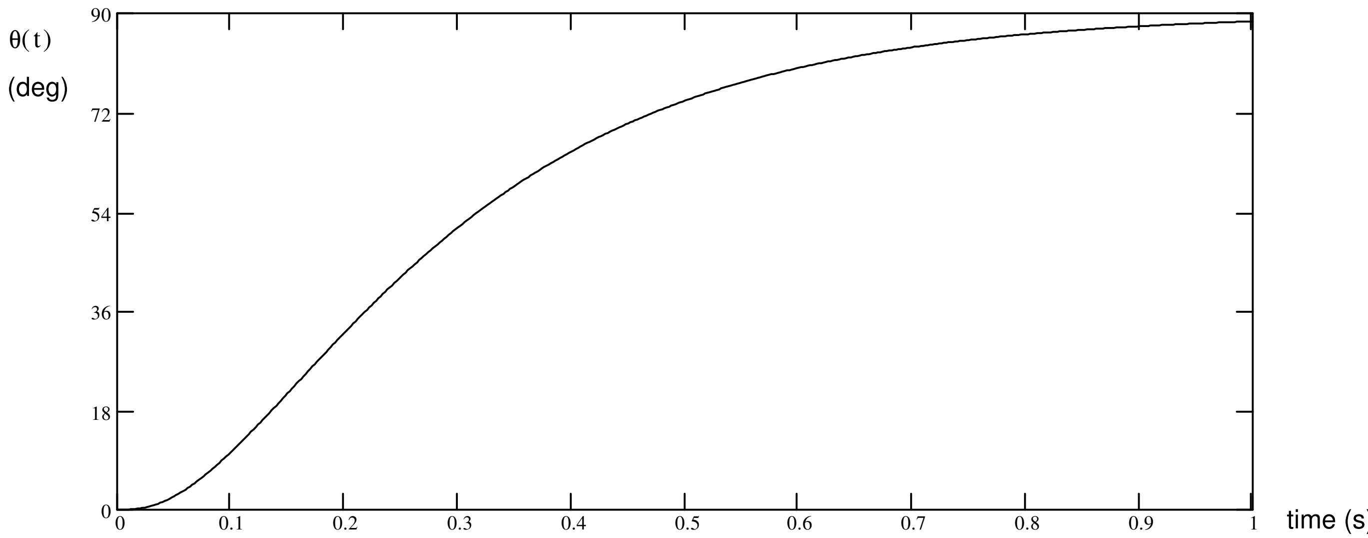
$r := \dfrac{K_p \cdot G \cdot K_T}{J \cdot L_a}$ $\quad a := q - \dfrac{p^2}{3}$ $\quad b := \dfrac{2 \cdot p^3}{27} - \dfrac{p \cdot q}{3} + r$ $\quad A := \sqrt[3]{-\dfrac{b}{2} + \sqrt{\dfrac{(b)^2}{4} + \dfrac{a^3}{27}}}$ $\quad B := \sqrt[3]{-\dfrac{b}{2} - \sqrt{\dfrac{(b)^2}{4} + \dfrac{a^3}{27}}}$

$s_1 := (A + B) - \dfrac{p}{3}$ $\quad s_1 = -4.83 \cdot sec^{-1}$ $\quad s_2 := -\dfrac{A+B}{2} - \dfrac{A-B}{2} \cdot \sqrt{-3} - \dfrac{p}{3}$ $\quad s_2 = -10.494 \cdot sec^{-1}$

$\theta_{fin} := 90 \cdot deg$ $\quad s_3 := -\dfrac{A+B}{2} + \dfrac{A-B}{2} \cdot \sqrt{-3} - \dfrac{p}{3}$ $\quad s_3 = -55.091 \cdot sec^{-1}$

$$\begin{pmatrix} D \\ E \\ F \end{pmatrix} := \begin{bmatrix} 1 & 1 & 1 \\ s_1 \cdot sec & s_2 \cdot sec & s_3 \cdot sec \\ (s_1)^2 \cdot sec^2 & (s_2)^2 \cdot sec^2 & (s_3)^2 \cdot sec^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\theta_{fin} \\ 0 \\ 0 \end{bmatrix} \qquad \begin{pmatrix} D \\ E \\ F \end{pmatrix} = \begin{pmatrix} -3.19 \\ 1.655 \\ -0.036 \end{pmatrix}$$

$$\theta(t) := \theta_{fin} + D \cdot e^{s_1 \cdot t} + E \cdot e^{s_2 \cdot t} + F \cdot e^{s_3 \cdot t}$$



for Gain = $G := 14$

$r := \dfrac{K_p \cdot G \cdot K_T}{J \cdot L_a}$ $\quad a := q - \dfrac{p^2}{3}$ $\quad b := \dfrac{2 \cdot p^3}{27} - \dfrac{p \cdot q}{3} + r$ $\quad A := \sqrt[3]{-\dfrac{b}{2} + \sqrt{\dfrac{(b)^2}{4} + \dfrac{a^3}{27}}}$ $\quad B := \sqrt[3]{-\dfrac{b}{2} - \sqrt{\dfrac{(b)^2}{4} + \dfrac{a^3}{27}}}$

$s_1 := (A + B) - \dfrac{p}{3}$ $\quad s_1 = -61.962 \cdot sec^{-1}$ $\quad s_2 := -\dfrac{A+B}{2} - \dfrac{A-B}{2} \cdot \sqrt{-3} - \dfrac{p}{3}$ $\quad s_2 = (-4.227 - 18.796j) \cdot sec^{-1}$

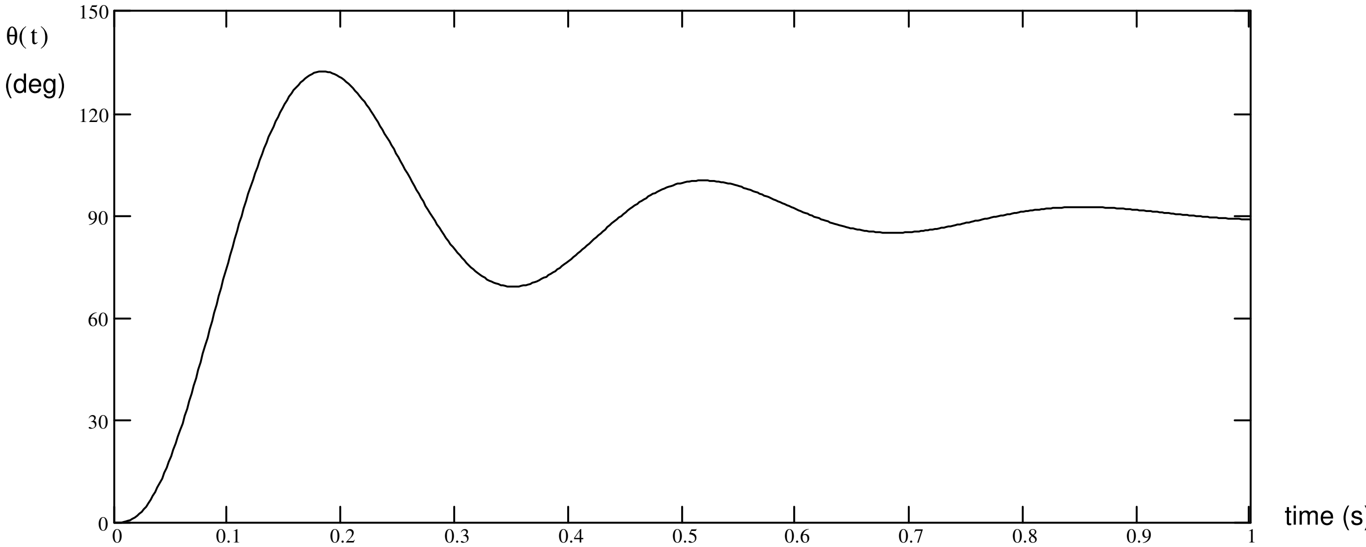
$\theta_{fin} := 90 \cdot deg$ $\quad s_3 := -\dfrac{A+B}{2} + \dfrac{A-B}{2} \cdot \sqrt{-3} - \dfrac{p}{3}$ $\quad s_3 = (-4.227 + 18.796j) \cdot sec^{-1}$

$$\begin{pmatrix} D \\ E \\ F \end{pmatrix} := \begin{bmatrix} 1 & 1 & 1 \\ s_1 \cdot sec & s_2 \cdot sec & s_3 \cdot sec \\ (s_1)^2 \cdot sec^2 & (s_2)^2 \cdot sec^2 & (s_3)^2 \cdot sec^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\theta_{fin} \\ 0 \\ 0 \end{bmatrix} \qquad \begin{pmatrix} D \\ E \\ F \end{pmatrix} = \begin{pmatrix} -0.158 \\ -0.706 - 0.419j \\ -0.706 + 0.419j \end{pmatrix}$$

$$\theta(t) := \theta_{fin} + D \cdot e^{s_1 \cdot t} + E \cdot e^{s_2 \cdot t} + F \cdot e^{s_3 \cdot t}$$

Plot on next page

ECE 3510 DC permanent-magnet motor, p3



G = 14, Decaying Oscillation (ringing)

for Gain = $G := 40$

$r := \frac{K_p \cdot G \cdot K_T}{J \cdot L_a}$ $\quad a := q - \frac{p^2}{3}$ $\quad b := \frac{2 \cdot p^3}{27} - \frac{p \cdot q}{3} + r$ $\quad A := \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{(b)^2}{4} + \frac{a^3}{27}}}$ $\quad B := \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{(b)^2}{4} + \frac{a^3}{27}}}$

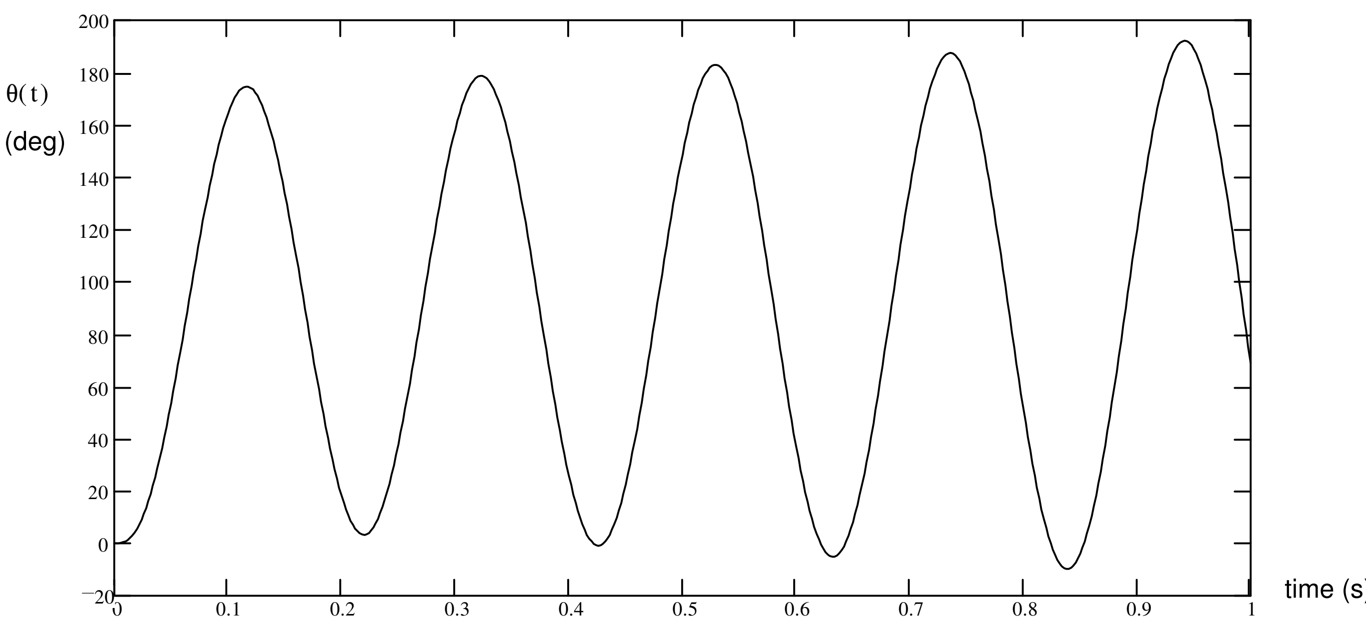
$s_1 := (A + B) - \frac{p}{3}$ $\quad s_1 = -70.87 \cdot sec^{-1}$ $\quad s_2 := -\frac{A+B}{2} - \frac{A-B}{2} \cdot \sqrt{-3} - \frac{p}{3}$ $\quad s_2 = 0.227 - 30.448j \cdot sec^{-1}$

$\theta_{fin} := 90 \cdot deg$ $\quad s_3 := -\frac{A+B}{2} + \frac{A-B}{2} \cdot \sqrt{-3} - \frac{p}{3}$ $\quad s_3 = 0.227 + 30.448j \cdot sec^{-1}$

$$\begin{pmatrix} D \\ E \\ F \end{pmatrix} := \begin{bmatrix} 1 & 1 & 1 \\ s_1 \cdot sec & s_2 \cdot sec & s_3 \cdot sec \\ (s_1)^2 \cdot sec^2 & (s_2)^2 \cdot sec^2 & (s_3)^2 \cdot sec^2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -\theta_{fin} \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} D \\ E \\ F \end{pmatrix} = \begin{pmatrix} -0.243 \\ -0.664 - 0.278j \\ -0.664 + 0.278j \end{pmatrix}$$

$\theta(t) := \theta_{fin} + D \cdot e^{s_1 \cdot t} + E \cdot e^{s_2 \cdot t} + F \cdot e^{s_3 \cdot t}$



G = 40, Growing Oscillation