ECE 3510 Bode Plot to Transfer Function Examples

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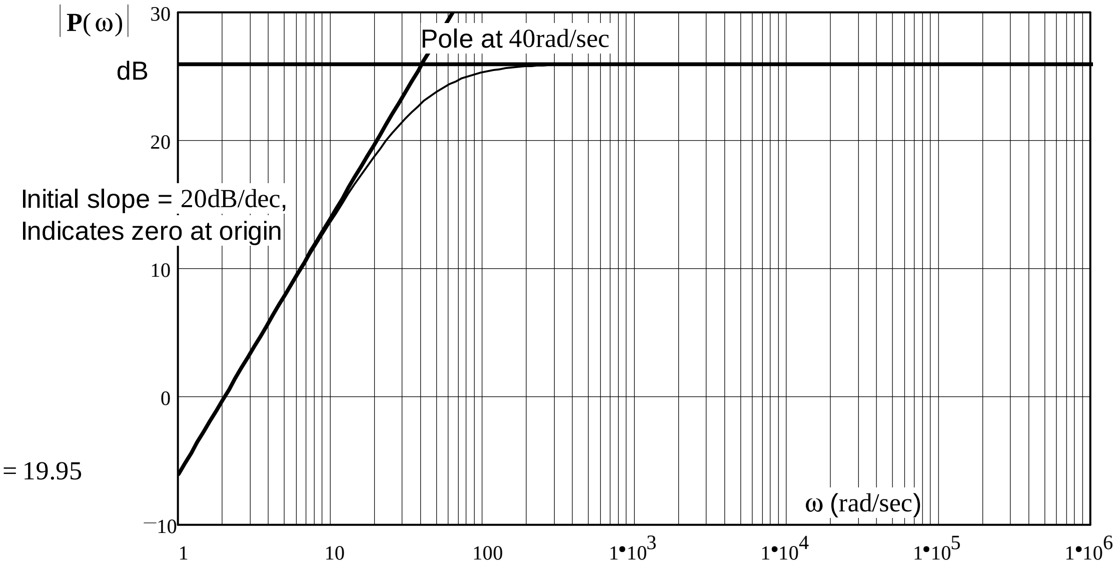
4/6/14, 4/4/17

Ex. 1 $\mathbf{P}(s) = ?$



$$\mathbf{P}(s) = K \cdot \frac{s}{s+40}$$

After 40rad/sec:

$$\mathbf{P}(s) \simeq K = 10^{\left(\frac{26 \cdot dB}{20 \cdot dB}\right)} = 19.95$$

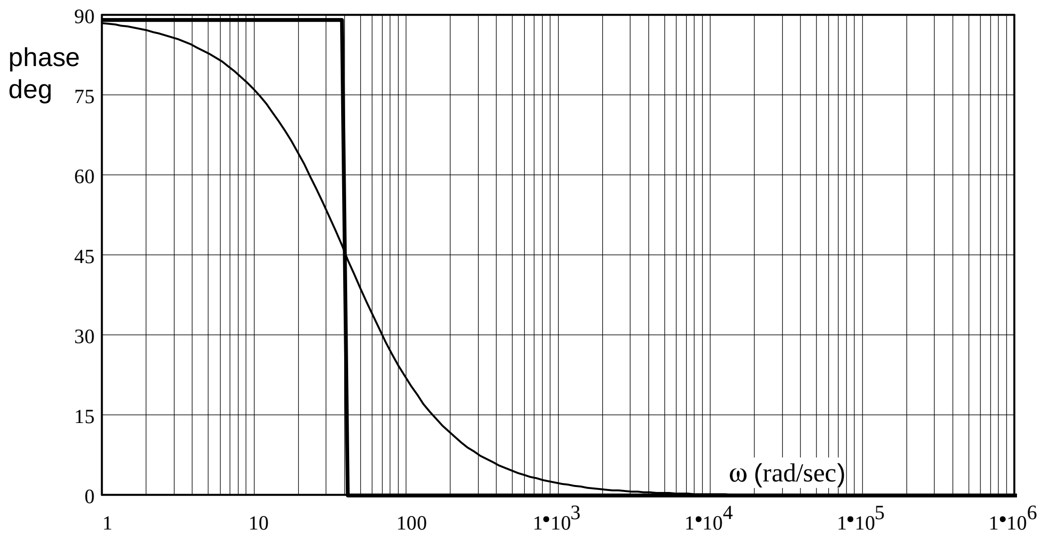
round to: $K := 20$

$$\arg\left(K \cdot \frac{j\omega}{\_ + 40}\right) = 90 \cdot deg$$

$$\arg\left(K \cdot \frac{j\omega}{j\omega + 0}\right) = 0 \cdot deg$$

Angles work out, so:

$$\mathbf{P}(s) = 20 \cdot \frac{s}{s+40}$$

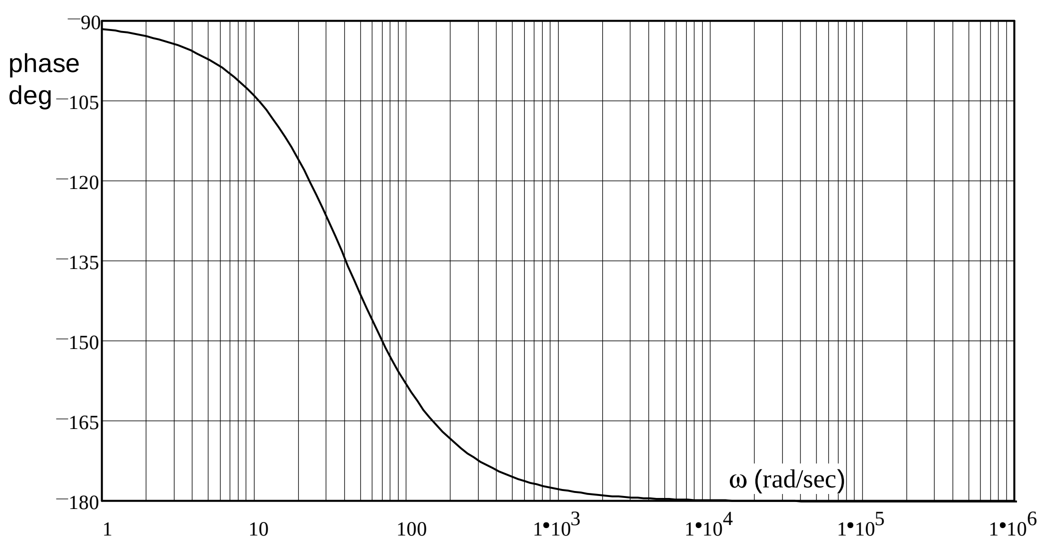


Ex. 2 What if the phase plot was:

$\mathbf{P}(s) = ?$

Angles are different by 180°, so:

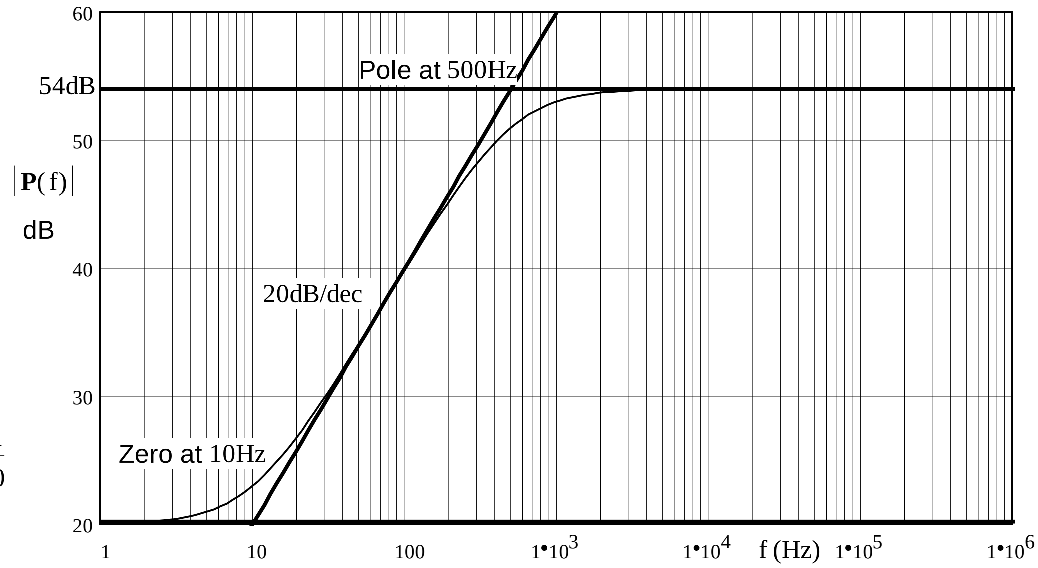
$$P(s) = -20 \cdot \frac{s}{s+40}$$



Ex. 3 $\mathbf{P}(s) = ?$

This plot is in Hz, which is what you would do in the lab with a scope or frequency counter.

$$\omega = 2\cdot\pi\cdot f$$



$$\mathbf{P}(s) = K\cdot\frac{s + 2\cdot\pi\cdot 10}{s + 2\cdot\pi\cdot 500}$$

At 1Hz:

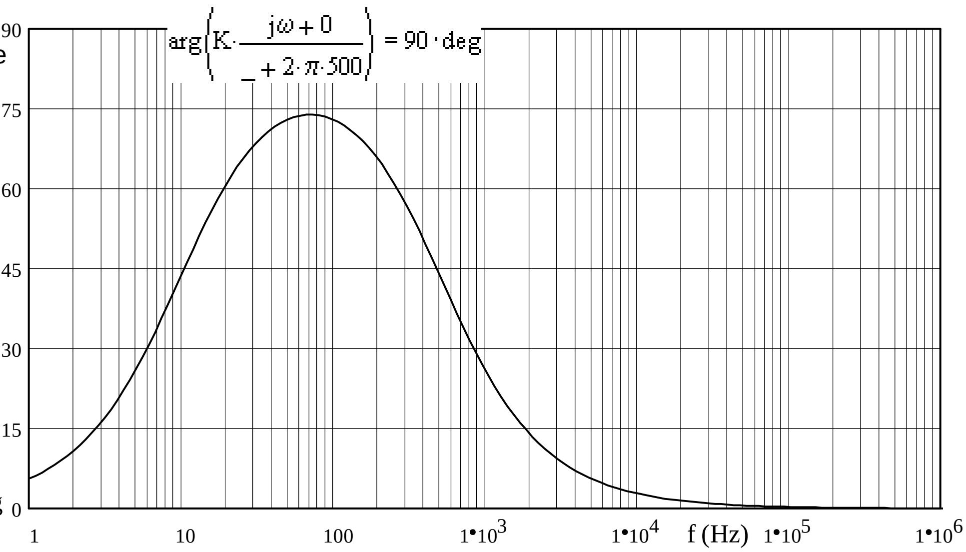
$$\mathbf{P}(s) \simeq K\cdot\frac{\_ + 2\cdot\pi\cdot 10}{\_ + 2\cdot\pi\cdot 500} = \frac{K}{50}$$

$$= 10^{\left(\frac{20\cdot dB}{20\cdot dB}\right)} = 10$$

$$K = 500$$

OR: $K = 10^{\left(\frac{54\cdot dB}{20\cdot dB}\right)} = 501.19$
(after 500Hz)

phase
deg



Angles work out, so:

$$\mathbf{P}(s) = 500\cdot\frac{s + 2\cdot\pi\cdot 10}{s + 2\cdot\pi\cdot 500}$$

$$= 500\cdot\frac{s + 62.83}{s + 3142}$$

$$\arg\left(K\cdot\frac{\_ + 2\cdot\pi\cdot 10}{\_ + 2\cdot\pi\cdot 500}\right) = 0\cdot deg$$

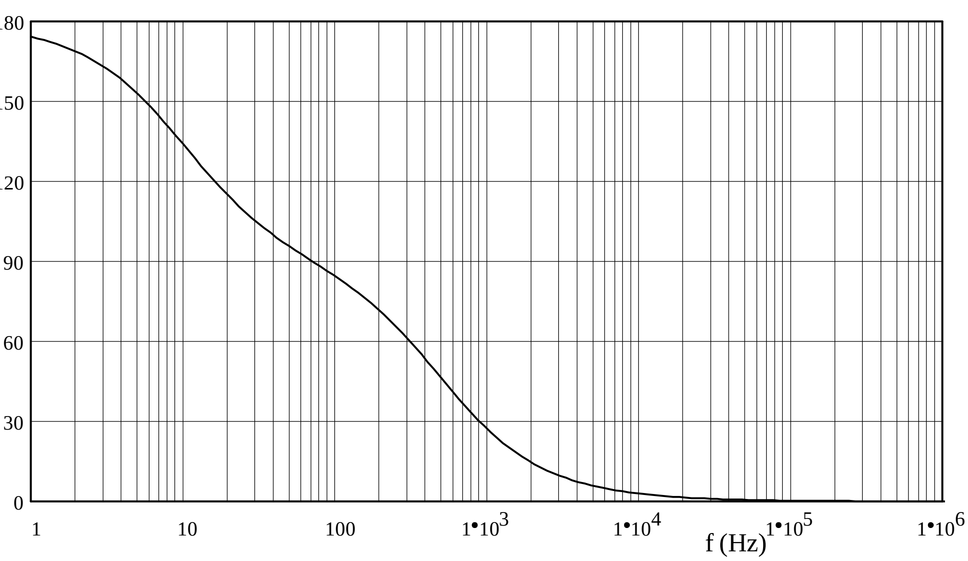
$$\arg\left(K\cdot\frac{j\omega + 0}{j\omega + 0}\right) = 0\cdot deg$$

Ex. 4 What if the phase plot was:

$\mathbf{P}(s) = ?$

Angle for 1st section is off by 180°. Angle for 2nd and 3rd sections are the same as above

deg
phase



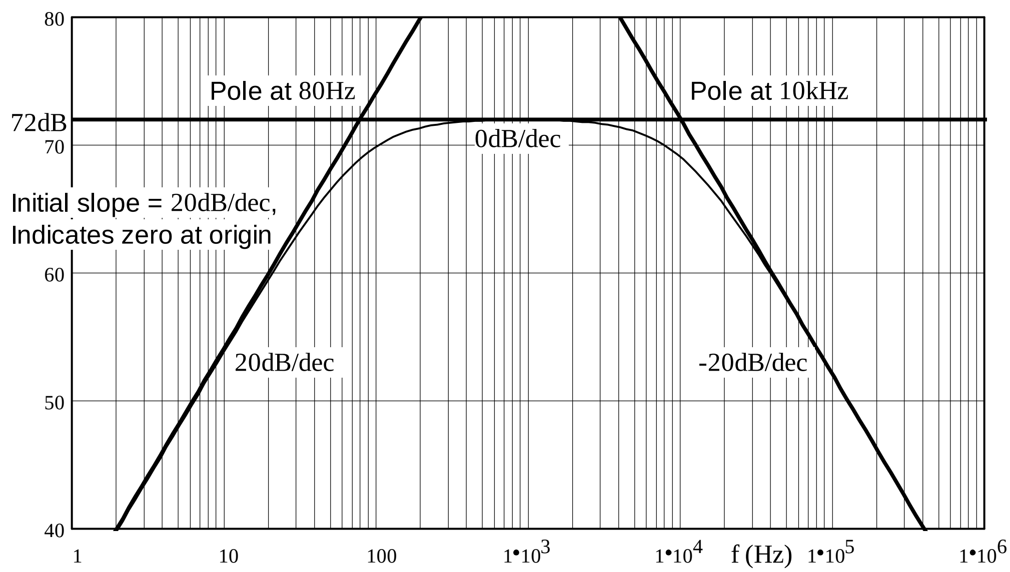
$$\arg\left(K\cdot\frac{\_ - 2\cdot\pi\cdot 10}{\_ + 2\cdot\pi\cdot 500}\right) = 180\cdot deg$$

$$\mathbf{P}(s) = 500\cdot\frac{s - 2\cdot\pi\cdot 10}{s + 2\cdot\pi\cdot 500}$$

$$= 500\cdot\frac{s - 62.83}{s + 3142}$$

Ex. 5 $\mathbf{P}(s) = ?$

$|\mathbf{P}(f)|$ dB

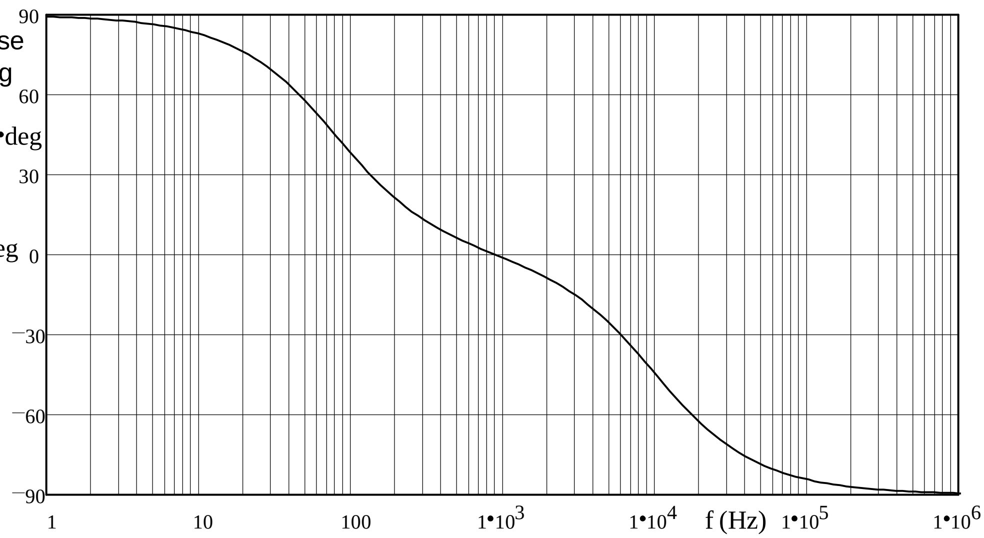


$$\mathbf{P}(s) = K \cdot \frac{s}{(s + 2\cdot\pi\cdot 80)\cdot(s + 2\cdot\pi\cdot 10000)}$$

From 80Hz to 10kHz:

$$\mathbf{P}(s) \simeq K \cdot \frac{j\omega}{(j\omega + 0)\cdot(\_ + 2\cdot\pi\cdot 10000)} = \frac{K}{2\cdot\pi\cdot 10000} = 10^{\left(\frac{72\cdot dB}{20\cdot dB}\right)} = 3981$$

$$K = 3981\cdot(2\cdot\pi\cdot 10000) = 2.5\cdot 10^{8}$$



$$\arg\left[\frac{j\omega}{(\_ + 2\cdot\pi\cdot 80)\cdot(\_ + 2\cdot\pi\cdot 10000)}\right] = 90\cdot deg$$

$$\arg\left[\frac{j\omega}{(j\omega + 0)\cdot(\_ + 2\cdot\pi\cdot 10000)}\right] = 0\cdot deg$$

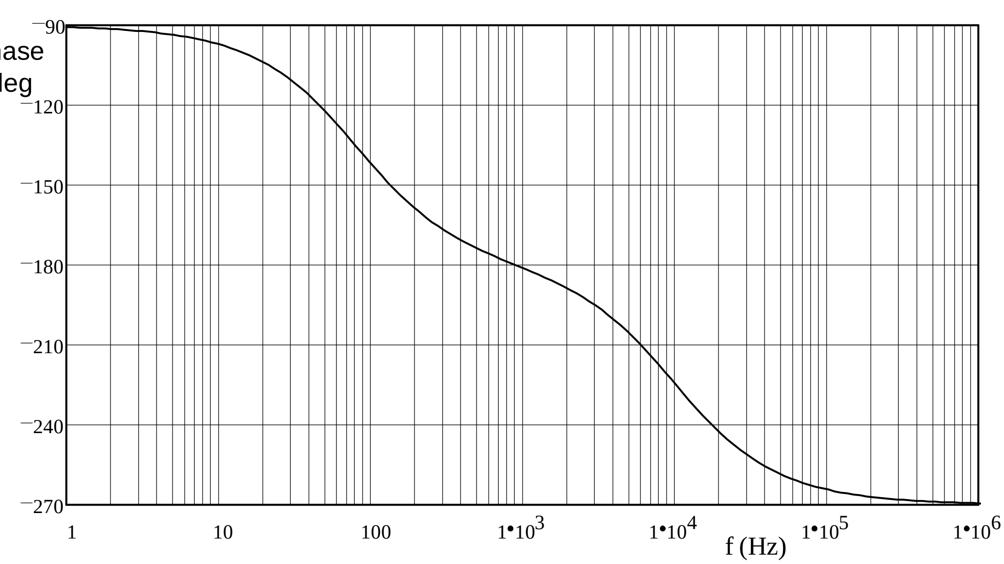
$$\arg\left[\frac{j\omega}{(j\omega + 0)\cdot(j\omega + 0)}\right] = -90\cdot deg$$

Angles work out, so:

$$\mathbf{P}(s) = \frac{25\cdot 10^{8}\cdot s}{(s + 2\cdot\pi\cdot 80)\cdot(s + 2\cdot\pi\cdot 10000)} = \frac{25\cdot 10^{8}\cdot s}{(s + 503)\cdot(s + 62832)}$$

Ex. 6 What if the phase plot was:

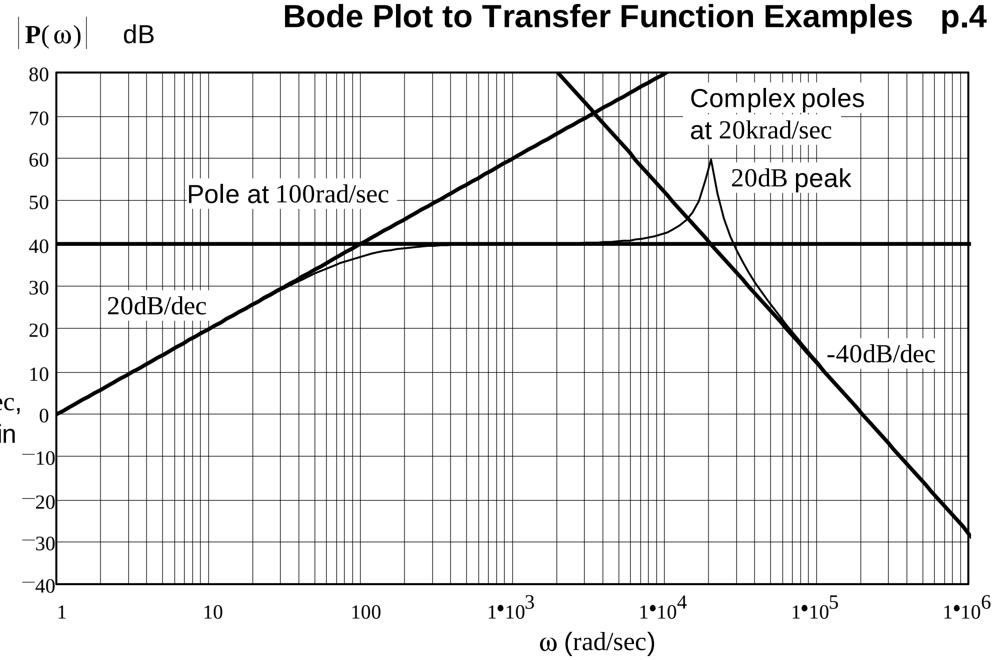
$\mathbf{P}(s) = ?$



Angles are different by 180º, so:

$$\mathbf{P}(s) = \frac{-25\cdot 10^{8}\cdot s}{(s + 503)\cdot(s + 62832)}$$

Ex. 7 $\mathbf{P}(s) = ?$



Initial slope = 20dB/dec,
Indicates zero at origin

$$\frac{K\cdot s}{(s+100)\cdot\left(s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}\right)}$$

complex poles at:

$$\omega_n := 20000\cdot\frac{rad}{sec}$$

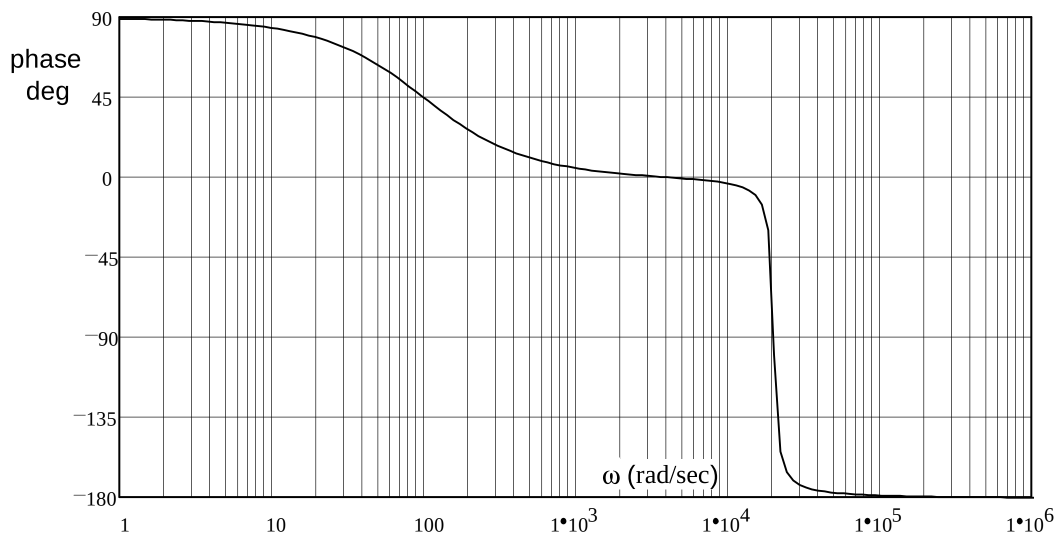
$$\omega_n^{\,2} = 4\bullet 10^8 \bullet\left(\frac{rad}{sec}\right)^2$$

peaking at complex zeros = 20dB, so:

$$10^{\frac{20}{20}} = 10 \quad = \quad \frac{1}{2\cdot\zeta}$$

$$\zeta := \frac{1}{2\cdot 10} \qquad \zeta = 0.05$$

$$2\cdot\zeta\cdot\omega_n = 2000\bullet\frac{1}{sec}$$



$$\mathbf{P}(s) = \frac{K\cdot s}{(s+100)\cdot\left(s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}\right)} = \frac{K\cdot s}{(s+100)\cdot\left(s^2+2000\cdot s+4\cdot 10^8\right)}$$

From 100 to 20krad/sec: $\mathbf{P}(s) \simeq \dfrac{K\cdot j\omega}{(j\omega+0)\cdot\left(\_^{2}+2000\cdot\_+4\cdot 10^8\right)} = \dfrac{K}{4\cdot 10^8} = 10^{\left(\frac{40\cdot dB}{20\cdot dB}\right)} = 100$

$$K = 100\cdot\left(4\cdot 10^8\right) = 4\bullet 10^{10}$$

Note: A somewhat more involved method is outlined in Nise section 10.13 (p.660 in 3rd ed., 665 in 4th). That method involves estimating only one pole or zero at a time and then subtracting the effect from original to more clearly see the others. This can work much better with real experimental data. Real data always has delay effects and other non-linearities which make the process much harder.