ECE 3510

PID Tuning

To convert to $G(s) = k_p + \frac{k_i}{s} + k_d s$ controller:

$k_p = k_p$

$k_i = \frac{k_p}{T_I}$

$K_d = K_p T_D$

▲ 4.4.2 Ziegler–Nichols Tuning of PID Regulators

As we will see in later chapters, sophisticated methods are available to develop a controller that will meet steady-state and transient specifications for both tracking input references and rejecting disturbances. These methods require that the designer have either a dynamic model of the process in the form of equations of motion or a detailed frequency response over a substantial range of frequencies. Either of these data can be quite difficult to obtain, and the difficulty has led to the development of sophisticated techniques of system model identification. Engineers early on explored ways to avoid these requirements.

Callender et al. (1936) proposed a design for the widely used PID controller by specifying satisfactory values for the controller settings based on estimates of the plant parameters that an operating engineer could make from experiments on the process itself. The approach was extended by J. G. Ziegler and N. B. Nichols (1942, 1943) who recognized that the step responses of a large number of process control systems exhibit a **process reaction curve** like that shown in Fig. 4.18, which can be generated from experimental step response data. The S-shape of the curve is characteristic of many systems and can be approximated by the step response of

Transfer function for a high-order system with a characteristic process reaction curve

$$\frac{Y(s)}{U(s)} = \frac{Ae^{-st_d}}{\tau s + 1}, \tag{4.109}$$

which is a first-order system with a time delay of t_d seconds. The constants in Eq. (4.109) can be determined from the unit step response of the process. If a tangent is drawn at the inflection point of the reaction curve, then the slope of the line is $R = A/\tau$ and the intersection of the tangent line with the time axis identifies the time delay $L = t_d$.

Ziegler and Nichols gave two methods for tuning the PID controller for such a model. In the first method the choice of controller parameters is designed to result in a closed-loop step response transient with a decay ratio of approximately 0.25.

Tuning by decay ratio of 0.25

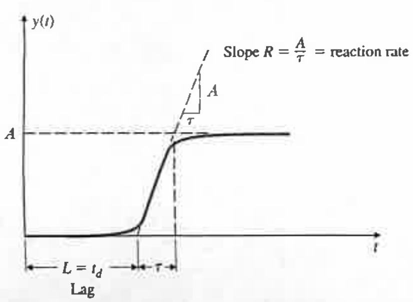


Figure 4.18
Process reaction curve

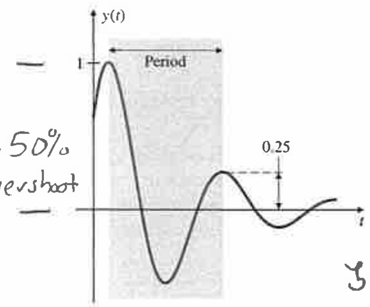
Open-loop.

Can only do this if open-loop is BIBO stable.

for plants w/o integrators or dominant complex poles

Feedback Control of Dynamic Systems, Franklin, Powell, etc

Figure 4.19
Quarter decay ratio



~50% overshoot

$\zeta = .21$, quite low.

This means that the transient decays to a quarter of its value after one period of oscillation, as shown in Fig. 4.19. A quarter decay corresponds to $\zeta = 0.21$ and is a reasonable compromise between quick response and adequate stability margins. The authors simulated the equations for the system on an analog computer and adjusted the controller parameters until the transients showed the decay of 25% in one period. The regulator parameters suggested by Ziegler and Nichols for the controller terms, defined by

$$D_c(s) = k_p(1 + \frac{1}{T_I s} + T_D s), \tag{4.110}$$

are given in Table 4.2.

TABLE 4.2

Ziegler–Nichols Tuning for the Regulator
$D(s) = k_p(1 + 1/T_I s + T_D s)$, for a decay ratio of 0.25

Type of Controller	Optimum Gain
Proportional	$k_p = 1/RL$
PI	$k_p = 0.9/RL$, $T_I = L/0.3$
PID	$k_p = 1.2/RL$, $T_I = 2L$, $T_D = 0.5L$

$T_I = \frac{k_p}{k_i}$

$T_D = \frac{k_d}{k_p}$

If open-loop is NOT BIBO then you must use the next method

Tuning by evaluation at limit of stability (ultimate sensitivity method)

In the **ultimate sensitivity method**, the criteria for adjusting the parameters are based on evaluating the amplitude and frequency of the oscillations of the system at the limit of stability rather than on taking a step response. To use the method, the proportional gain is increased until the system becomes marginally stable and continuous oscillations just begin, with amplitude limited by the saturation of the actuator. The corresponding gain is defined as K_u

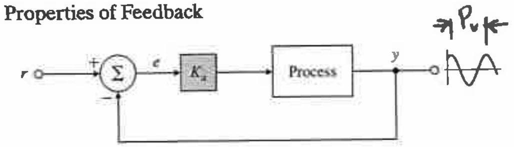


Figure 4.20
Determination of the ultimate gain and period

(called the **ultimate gain**) and the period of oscillation is P_u (called the **ultimate period**). These are determined as shown in Figs. 4.20 and 4.21. P_u should be measured when the amplitude of oscillation is as small as possible. Then the tuning parameters are selected as shown in Table 4.3.

TABLE 4.3

Ziegler–Nichols Tuning for the Regulator $D_c(s) = k_p(1 + 1/T_I s + T_D s)$, Based on the Ultimate Sensitivity Method

Type of Controller	Optimum Gain	
Proportional	$k_p = 0.5K_u$	
PI	$\begin{cases} k_p = 0.45K_u, \\ T_I = \frac{P_u}{1.2} \end{cases}$	$= \frac{k_p}{k_i}$
PID	$\begin{cases} k_p = 0.6K_u, \\ T_I = \frac{1}{2}P_u, \\ T_D = \frac{1}{8}P_u \end{cases}$	$= \frac{k_d}{k_p}$

Experience has shown that the controller settings according to Ziegler–Nichols rules provide acceptable closed-loop response for many systems. The process operator will often do final tuning of the controller iteratively on the actual process to yield satisfactory control.[10]

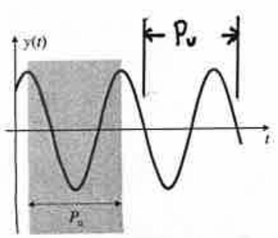


Figure 4.21
Neutrally stable system

[10] Tuning of PID controllers has been the subject of continuing study since 1936. A modern publication on the topic is H. Panagopoulous, K. J. Åström, and T. Hagglund, *Proceedings of the American Control Conference*, San Diego, CA, June 1999.

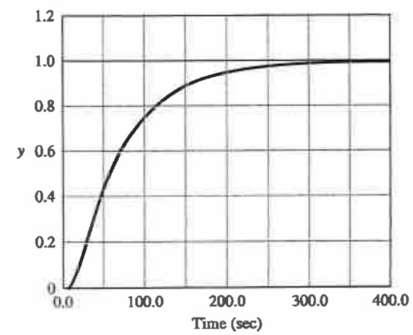


Figure 4.22
A measured process reaction curve

EXAMPLE 4.9

Tuning of a Heat Exchanger: Quarter Decay Ratio

Consider the heat exchanger of Example 2.13. The process reaction curve of this system is shown in Fig. 4.22. Determine proportional and PI regulator gains for the system using the Zeigler–Nichols rules to achieve a quarter decay ratio. Plot the corresponding step responses.

Solution. From the process reaction curve, we measure the maximum slope to be $R \cong \frac{1}{90}$ and the time delay to be $L \cong 13$ sec. According to the Zeigler–Nichols rules of Table 4.2 the gains are as follows:

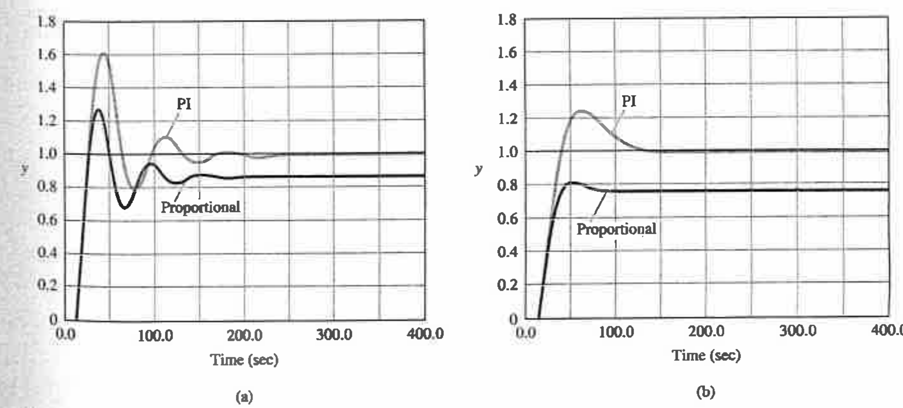


Figure 4.23 Closed-loop step responses

$$\text{Proportional}: k_p = \frac{1}{RL} = \frac{90}{13} = 6.92,$$

$$\text{PI}: k_p = \frac{0.9}{RL} = 6.22 \quad \text{and} \quad T_I = \frac{L}{0.3} = \frac{13}{0.3} = 43.3$$

Figure 4.23(a) shows the step responses of the closed-loop system to these two regulators. Note that the proportional regulator results in a steady-state offset, while the PI regulator tracks the step exactly in the steady-state. Both regulators are rather oscillatory and have considerable overshoot. If we arbitrarily reduce the gain k_p by a factor of 2 in each case, the overshoot and oscillatory behaviors are substantially reduced, as shown in Fig. 4.23(b).

EXAMPLE 4.10

Tuning of a Heat Exchanger: Oscillatory Behavior

Proportional feedback was applied to the heat exchanger in the previous example until the system showed nondecaying oscillations in response to a short pulse (impulse) input, as shown in Fig. 4.24. The ultimate gain was $K_u = 15.3$, and the period was measured at $P_u = 42$ sec. Determine the proportional and PI regulators according to the Zeigler–Nichols rules based on the ultimate sensitivity method. Plot the corresponding step responses.

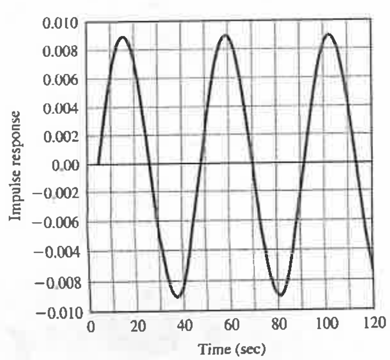
Solution. The regulators from Table 4.3 are

$$\text{Proportional}: k_p = 0.5K_u = 7.65,$$

$$\text{PI}: k_p = 0.45\,K_u = 6.885 \quad \text{and} \quad T_I = \frac{1}{1.2}P_u = 35.$$

The step responses of the closed-loop system are shown in Fig. 4.25(a). Note that the responses are similar to those in Example 4.9. If we reduce k_p by 50%, then the overshoot is substantially reduced, as shown in Fig. 4.25(b).

Figure 4.24
Ultimate period of heat exchanger



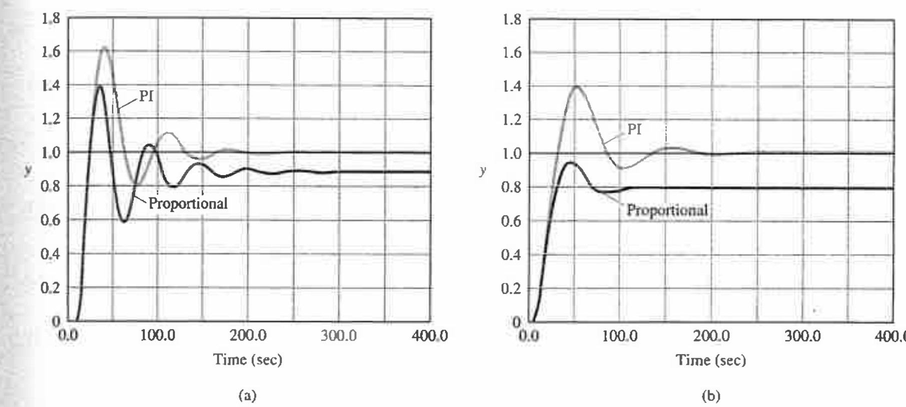


Figure 4.25 Closed-loop step response

▲ 4.4.3 Truxal's Formula for the Error Constants

In this chapter we have derived formulas for the error constants in terms of the system transfer function. The most common case is the type 1 system, whose error constant is K_v, the velocity error constant. Truxal (1955) derived a formula for the velocity constant in terms of the closed-loop poles and zeros, a formula that connects the steady-state error to the dynamic response. Since control design often requires a trade-off between these two characteristics, Truxal's formula can be useful to know. Its derivation is quite direct. Suppose the closed-loop transfer function $\mathcal{T}(s)$ of a type 1 system is

$$\mathcal{T}(s) = K\frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}. \tag{4.111}$$

Since the steady-state error in response to a step input in a type 1 system is zero, the DC gain is unity. Thus,

$$\mathcal{T}(0) = 1. \tag{4.112}$$

The system error is given by

$$E(s) \triangleq R(s) - Y(s) = R(s)\left[1 - \frac{Y(s)}{R(s)}\right] = R(s)[1 - \mathcal{T}(s)]. \tag{4.113}$$

How do the PID parameters affect system dynamics?

Use K_p to decrease rise time. (increase speed)
Use K_D to reduce overshoot & settling time.
Use K_i to eliminate steady-state error

The effects of increasing each of the controller parameters K_P, K_I and K_D can be summarized as

1st time from
from 10% to 90%

Response	Rise Time	Overshoot	Settling Time	S-S Error
K_P	Decrease	Increase	NT	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	NT	Decrease	Decrease	NT

(Allows greater K_p)

NT: No definite trend. Minor change.
You may want to take notes of this table. It will be useful in the later part of the lesson.