

ECE 3510 Root Locus Design Crib Sheet

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Using 2nd-order approximation: $\frac{N(s)}{(s+a)^2 + b^2} = \frac{N(s)}{s^2 + 2 \cdot a \cdot s + a^2 + b^2} = \frac{N(s)}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2}$

$\omega_n^2 = a^2 + b^2$ $\omega_n = \text{natural frequency}$

$\zeta \cdot \omega_n = a$

$\zeta = \frac{a}{\omega_n} = \frac{a}{\sqrt{a^2 + b^2}} = \text{damping factor}$ $\zeta = \sin\left(\text{atan}\left(\frac{a}{b}\right)\right)$

Overshoot: $OS = e^{-\frac{\pi \cdot a}{b}}$ $\%OS = 100\% \cdot e^{-\frac{\pi \cdot a}{b}}$ $\frac{a}{b} = \frac{\ln(OS)}{-\pi}$

angle of constant damping line: $90 \cdot \text{deg} + \text{atan}\left(\frac{a}{b}\right)$

2% settling time: $T_s = \frac{4}{a} = \frac{4}{\zeta \cdot \omega_n}$

Time of first peak: $T_p = \frac{\pi}{b}$

Static error constant (position): $K_p = \lim_{s \rightarrow 0} K \cdot C(s) \cdot G(s)$ $e_{\text{step}}(\infty) = e_{\text{step}} = \frac{1}{1 + K_p}$ Nise p378

Lag compensation improves K_p , K_v and K_a by $\frac{z_c}{p_c}$ IE: $K_{pc} \approx K_{puc} \cdot \frac{z_c}{p_c}$

Searching along a line of constant damping:

Try s values, choosing b: $s = -\left(\frac{a}{b} \cdot b\right) + b \cdot j$ Test: $\arg(G(s)) \pm 180^\circ$ or $-\text{Re}(G(s)) \gg \text{Im}(G(s))$

Linear interpolation: new b = $b_1 - \frac{b_2 - b_1}{\text{Im}(G(s_2)) - \text{Im}(G(s_1))} \cdot \text{Im}(G(s_1))$

Can also try "a" values with slight modification of the above.

Weird forms from Nise book:

$\sigma_d = a$ $\%OS = 100\% \cdot e^{\frac{-\zeta \cdot \pi}{\sqrt{1 - \zeta^2}}}$ p195

$\omega_d = b$ $\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + (\ln(OS))^2}}$ p195 $T_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$ p194

p378 Static error constant (ramp): $K_v = \lim_{s \rightarrow 0} s \cdot K \cdot C(s) \cdot G(s)$ $e_{\text{ramp}} = \frac{1}{K_v}$
(velocity)

Static error constant (parabola): $K_a = \lim_{s \rightarrow 0} s^2 \cdot K \cdot C(s) \cdot G(s)$ $e_{\text{parabola}} = \frac{1}{K_a}$
(acceleration)