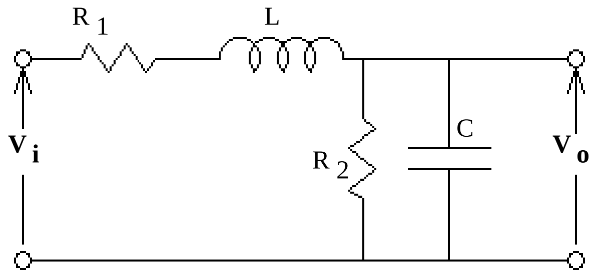
ECE 3510 Input and Output Impedance

A.Stolp 4/10/22 a

Let's find the transfer function of this circuit.



$\mathbf{H}(s) = ?$

$$\mathbf{H}(s) = \frac{\mathbf{V_o}(s)}{\mathbf{V_i}(s)} = \frac{\dfrac{1}{\dfrac{1}{R_2}+C\cdot s}}{R_1 + L\cdot s + \dfrac{1}{\dfrac{1}{R_2}+C\cdot s}} \quad \frac{\dfrac{1}{R_2}+C\cdot s}{\dfrac{1}{R_2}+C\cdot s}$$

$$= \frac{1}{R_1\cdot\left(\dfrac{1}{R_2}+C\cdot s\right) + L\cdot s\cdot\left(\dfrac{1}{R_2}+C\cdot s\right) + 1}$$

$$= \frac{1}{\dfrac{R_1}{R_2} + R_1\cdot C\cdot s + \dfrac{L\cdot s}{R_2} + L\cdot s\cdot C\cdot s + 1} \quad \frac{\dfrac{1}{L\cdot C}}{\dfrac{1}{L\cdot C}} \qquad = \frac{\dfrac{1}{L\cdot C}}{\dfrac{R_1}{R_2}\cdot\dfrac{1}{L\cdot C} + \dfrac{R_1\cdot C}{L\cdot C}\cdot s + \dfrac{L\cdot s}{R_2}\cdot\dfrac{1}{L\cdot C} + s^2 + \dfrac{1}{L\cdot C}}$$

$$= \frac{\dfrac{1}{L\cdot C}}{s^2 + \left(\dfrac{R_1}{L} + \dfrac{1}{R_2\cdot C}\right)\cdot s + \left(1 + \dfrac{R_1}{R_2}\right)\cdot\dfrac{1}{L\cdot C}}$$

Input Impedance

What load does this circuit place on the source of $\mathbf{V_i}$?

$$\mathbf{Z_{in}}(s) = R_1 + L\cdot s + \frac{1}{\dfrac{1}{R_2}+C\cdot s}$$

OR, if this circuit is followed by another circuit with $\mathbf{Z_{in2}}(s)$, then,

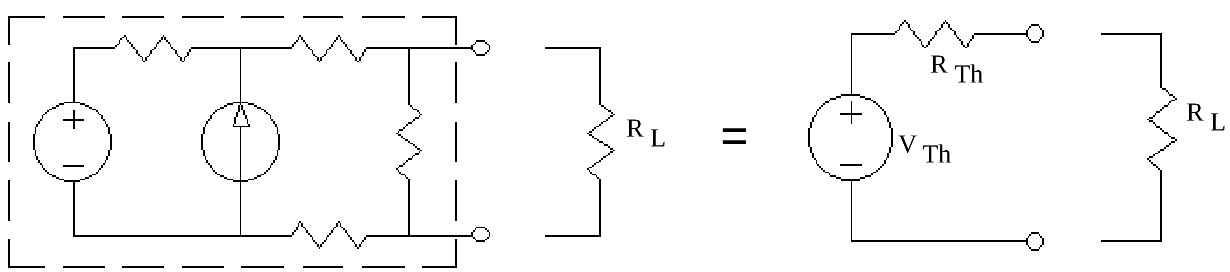
$$\mathbf{Z_{in}}(s) = R_1 + L\cdot s + \frac{1}{\dfrac{1}{R_2}+C\cdot s + \dfrac{1}{\mathbf{Z_{in2}}(s)}}$$

Usually, the higher the input impedance, the better.

Output Impedance

Output Impedance is just like the Thévenin Resistance

Thévevin Equivalent Circuit

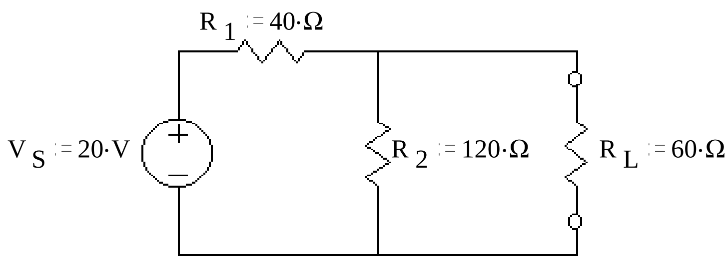


Thévenin equivalent

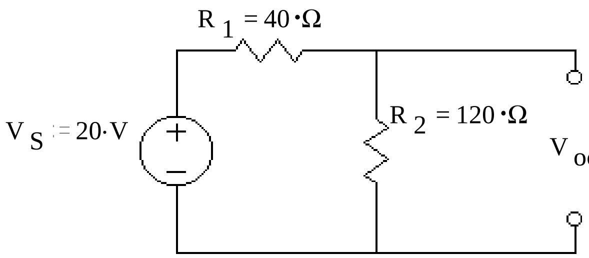
To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage (V_{Th}).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
4) Draw the Thévenin equivalent circuit and add your values.

Ex 1 Find the Thévenin equivalent:



To find a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage (V_{Th}).

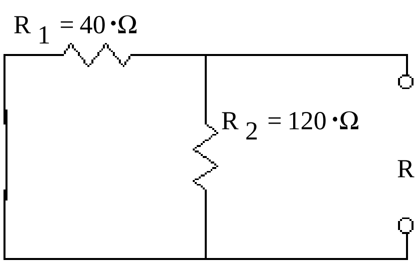


Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S \cdot \frac{R_2}{R_1 + R_2} \qquad V_{Th} = 15 \cdot V$$

2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

Zero the source



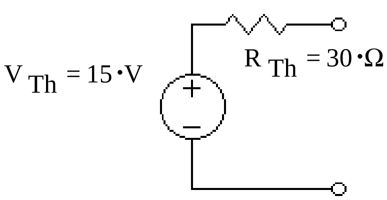
3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.)
This is the Thévenin source resistance (R_{Th}).

Find the Thevenin resistance:

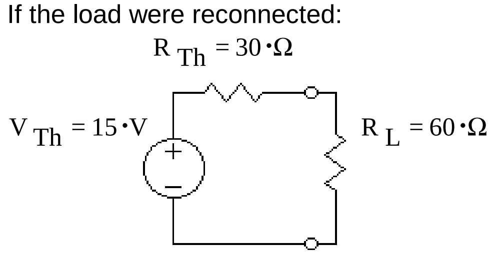
$$R_{Th} := \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} \qquad R_{Th} = 30 \cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:



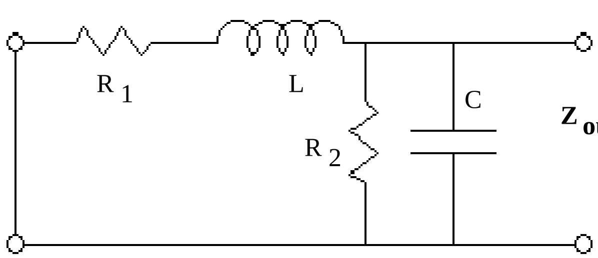
If the load were reconnected:



$$V_L = V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} = 10 \cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7 \cdot mA$$

Output Impedance



$$\mathbf{Z}_{\mathbf{out}}(s) = \frac{1}{\dfrac{1}{R_1 + L \cdot s} + \dfrac{1}{R_2} + C \cdot s} \left(\frac{R_1 + L \cdot s}{R_1 + L \cdot s} \right)$$

$$\mathbf{Z}_{\mathbf{out}}(s) = \frac{R_1 + L \cdot s}{1 + \dfrac{R_1}{R_2} + \dfrac{L}{R_2} \cdot s + C \cdot s \cdot R_1 + L \cdot C \cdot s^2} \frac{\left(\dfrac{1}{L \cdot C} \right)}{\left(\dfrac{1}{L \cdot C} \right)} = \frac{\dfrac{1}{C} \cdot s + \dfrac{R_1}{L \cdot C}}{s^2 + \left(\dfrac{1}{C \cdot R_2} + \dfrac{R_1}{L} \right) \cdot s + \dfrac{1}{L \cdot C} \cdot \left(1 + \dfrac{R_1}{R_2} \right)}$$

OR, if this circuit is preceded by another circuit with $\mathbf{Z}_{\mathbf{out0}}(s)$, then,

$$\mathbf{Z}_{\mathbf{out}}(s) = \frac{1}{\dfrac{1}{\mathbf{Z}_{\mathbf{out0}}} + \dfrac{1}{R_1 + L \cdot s} + \dfrac{1}{R_2} + C \cdot s}$$

Usually, the lower the output impedance, the better