Sample Final Exam with Solutions

Name ______________________________

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UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ANTENNA THEORY AND DESIGN

ECE 5324/6234

FINAL EXAMINATION

April 30, 2009

For students taking a two-hour final examination, **do all four problems**
(maximum score = 110 points)

For students taking a one-hour final examination, **do Problems 1, 2, and 3**
(maximum score = 80 points)

1. (30 points)

pts

10 a. Calculate the gain in dBi in the axial direction ($\phi = 0$) for a half-wave dipole that is placed axially at a distance of 0.3λ from the corner of a 90° corner reflector antenna at a frequency of 400 MHz. For this antenna, assume a feed point impedance $Z_1 = 47 + j100\Omega$.

You may be able to compare the gain that you get with the gain for this antenna given on page 57 of the Class Notes (which unfortunately is in dBd rather than dBi).

Hints: Use Eq. 5-48 of the Text to obtain max value of AF. Note that for an antenna system (from pp. 32, 33 fo Class Notes)

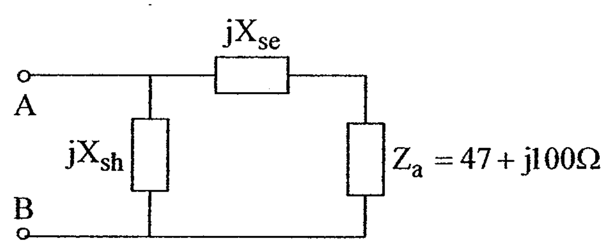
$$\text{Gain}\Big|_{\text{dBd}} = 10\log\left\{\frac{R_{Ao}}{R_{Ai}}\,|AF|^2_{max}\right\}$$

Convert the gain in dBd to dBi.

5 b. Calculate the effective A_e if this antenna were used as a receiving antenna.

5 c. Calculate the length of the dipole and the distances of the antenna from the corner of the 90° corner reflector.

10 d. Design an inverted-L matching circuit of the following configuration to conjugate-match this antenna to a source of impedance $Z_s = 100 + j75\Omega$



Note that for conjugate matching, we need $Z_{AB} = Z_s^* = 100 - j75\Omega$.

Calculate the values of the reactances jX_{se} and jX_{sh}.

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1. a. It is given that $Z_i = 47 + j\,100\ \Omega$

Thus $R_i = 47\ \Omega$

For a 90° corner reflector antenna, from Eq. 5-48

$$AF(\theta = 90°, \phi = 0) = 2\cos(\beta s \cos\phi)\Big|_{\phi=0} - 2\cos(\beta s \sin\phi)\Big|_{\phi=0}$$

$$= 2\cos(\beta s) - 2 = -2.618$$

since $\beta s = \frac{2\pi}{\lambda} \times 0.3\lambda = 0.6\pi$; $\cos(0.6\pi) = -0.309$

$$\text{Gain}\Big|_{dBd} = 10\log\left(\frac{73}{47} \times (2.618)^2\right) = 10\log(10.645)$$
$$= 10.27\ dBd$$

This may be compared to a gain of approximately 10.1 – 10.2 dBd from the graph on p. 57 of class Notes. The slight difference in the calculated gain is due to the difficulty in impedance Z_i from the graphs for real and imaginary parts of the impedance Z_i from p. 58 of the class Notes.

$$G \text{ in } dBi = 10.27 + \text{gain of half wave dipole} = 10.27 + 2.15 = 12.42\ dBi$$

(gain of half wave dipole = $10\log 1.64$ (see p. 6 of class Notes))

b. $$\frac{4\pi A_e}{\lambda^2} = \text{Gain} = 10^{12.42/10} = 10^{1.242} = 1.64 \times 10.645 = 17.46$$

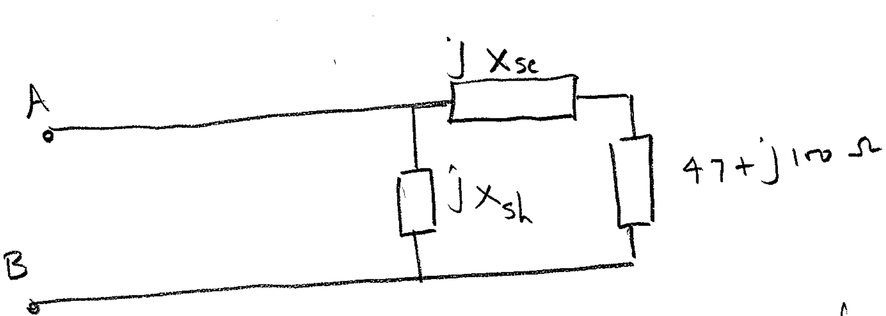
$\lambda|_{400\,MHz}$

$$A_e = \frac{(75)^2}{4\pi} \times 17.46 = \boxed{0.782\ m^2}$$

c. length of the dipole $= 0.5\lambda = \frac{75}{2} = 37.5$ cm

Spacing s to the corner of the 90° reflector $= 0.3\lambda = \boxed{22.5\text{ cm}}$

d.



Using a procedure similar to that on p. 56 of class Notes

$$Y_{AB} = \frac{1}{100 - j75} = \frac{100 + j75}{(100)^2 + (75)^2} = \frac{1}{47 + j(X_{se}+100)} + \frac{1}{jX_{sh}} \quad (1)$$

Equating real and imaginary parts of the equation on both sides of Eq. (1), we can write

$$\frac{47}{(47)^2 + (X_{se}+100)^2} = \frac{100}{15625} = \frac{1}{156.25} \quad (2)$$

$$47 \times 156.25 = (47)^2 + (X_{se}+100)^2 \quad (3)$$

$$X_{se} + 100 = \pm\sqrt{47 \times 109.25} = \pm 71.66 \quad (4)$$

$$X_{se} = \boxed{-28.34}\,;\ -171.66 \quad (5)$$

We take the smaller value of X_{se} (because it will vary less as frequency changes)

$$X_{se} = -28.34$$

Equating the imaginary parts on both sides of Eq. (1), we can write

$$\frac{-j(\overset{71.66}{\cancel{X_{se}+100}})}{(47)^2 + (X_{se}+100)^2} = \frac{-j\,71.66}{47 \times 156.25} = \frac{-j}{X_{sh}}$$

(from Eq. (3))

$$X_{sh} = \frac{47 \times 156.25}{71.66} = \boxed{102.48\ \Omega}$$

2. (20 points)

A helical antenna to be used as a radiator at 1.65 GHz has the following specifications:

Diameter of the helix = 2.25″
Number of turns = 10
Spacing S (pitch) = 0.8″

pts

4 a. For this antenna, calculate C/λ.

For the axial mode of radiation, which is typical for this antenna, calculate:

4 b. gain in decibels.
4 c. input resistance.
4 d. half-power beam width in degrees.
4 e. axial ratio for the radiated fields.

2. The material on the axial mode of a helical antenna $\left(\frac{3}{4} < C/\lambda < \frac{4}{3}\right)$ is discussed on pp. 235 – 239 of the Stutzman and Thiele Text

a. For this antenna $\frac{C}{\lambda} = \frac{\pi D}{\lambda} = \frac{\pi \times 2.25 \times 2.54}{30/1.65} = 0.987$

Note that $\lambda = 30/1.65 = 18.18$ cm

b. From Eq. 6-34 on p. 237 of the Text

$$G = 6.2 \left(\frac{C}{\lambda}\right)^2 \frac{NS}{\lambda} = \frac{6.2 \times (0.987)^2 \times 10 \times 0.8 \times 2.54}{18.18} = 6.75$$

$$G\Big|_{dB} = 10 \log 6.75 = \boxed{8.29\ dB}$$

c. Input resistance from Eq. 6-36 on p. 238 of the Text

$$R_A = 140 \times \frac{C}{\lambda} = 140 \times 0.987 = \boxed{138.2\ \Omega}$$

d. HPBW from Eq. 6-33 of the Text

$$HP = \frac{65^\circ}{0.987\sqrt{\frac{10 \times 0.8 \times 2.54}{18.18}}} = \boxed{62.29^\circ}$$

e. From Eq. 6-35 of the Text

$$\text{Axial Ratio} = |AR| = \frac{2N+1}{2N} = \frac{21}{20} = \boxed{1.05}$$

3. (30 points)

Design a parabolic antenna with a gain of 40 dBi at 12.5 GHz. For this antenna, assume n = 2 parabolic taper on a pedestal with edge illumination that is -14 dB relative to the illumination at the center. In particular, calculate the following:

Pts		
5	a.	the diameter of the antenna.
4	b.	half-power beamwidth in degrees.
5	c.	effective area of the antenna.
4	d.	side lobe level relative to the major lobe.
6	e.	the maximum power density S_{max} at a distance of 35,600 km (for a synchronous satellite) if the radiated power = 100 W.
6	f.	power received by the antenna of the satellite, given that the gain of the satellite antenna is 35 dBi and attenuation of the atmosphere is 2 dB.

3. a. From p. 320 of the Text (Table 7-1 part b)

for $n = 2$ parabolic taper on a pedestal for edge illumination of -14 dB relative to the illumination at the center

$$\epsilon_t = 0.792$$

$$\text{Gain} = 0.792\left(\frac{\pi D}{\lambda}\right)^2 = 10^4 \quad (40\ \text{dBi})$$

$$\lambda = \frac{30}{12.5} = 2.4\ \text{cm}$$

$$D = \sqrt{\frac{10^4}{0.792}} \times \frac{2.4}{\pi} = 85.84\ \text{cm}$$

b. $$HP = 1.23\frac{\lambda}{D}\ \text{rad} = 1.97^\circ$$

c. $$\frac{4\pi A_e}{\lambda^2} = \text{Gain} = 10^4$$

$$A_e = 0.792\ \frac{\pi D^2}{4} = 0.792\ A_{physical} = 4583.5\ \text{cm}^2$$

or $\boxed{0.4583\ \text{m}^2}$

d. Side lobe level $= -31.7$ dB relative to the major lobe

f. From Eq. 2-99 on p. 80 of the Text

$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t(\text{dB}) + G_r(\text{dB}) - 20\log R(\text{Km}) - 20\log f_{MHz} - 32.44 - \underset{L_{dB}}{\text{loss of the atmosphere in dB}}$$

$$= 50 + 40 + 35 - \underbrace{20\log(35{,}600)}_{91.03} - \underbrace{20\log(12{,}500)}_{81.94} - 32.44 - 2$$

$$= 125 - 91.03 - 81.94 - 34.44 = -82.41\ \text{dBm}$$

$$= 0.574 \times 10^{-11}\ \text{W} = 5.74 \times 10^{-12}\ \text{W or } 5.74\ \text{pW}$$

e. From Eq. 2-92 of the Text

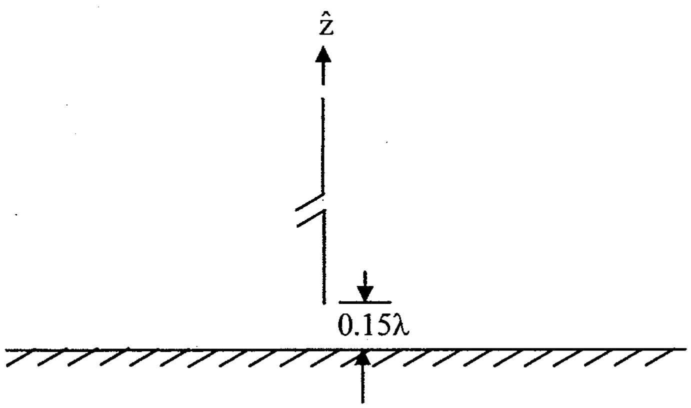
$$S = \frac{P_t G_t}{4\pi R^2} \times 10^{-0.2} = \frac{10^2 \times 10^4}{4\pi \times (3.56 \times 10^7)^2} \times 10^{-0.2}$$

($10^{-0.2}$ ← 2 dB loss of the atmosphere)

$$= 6.28 \times 10^{-11} \times 10^{-0.2} = \boxed{3.96 \times 10^{-11}\ \text{W/m}^2}$$

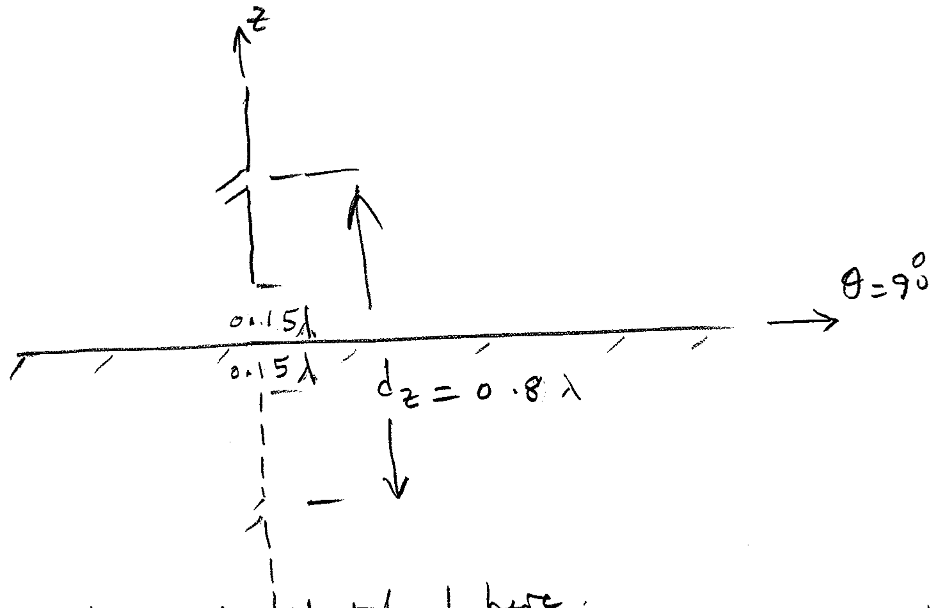
4. (30 points)

A vertical 0.4781λ (nominal **half-wave**) dipole is placed above ground as shown below



pts		
4	a.	Draw the image antenna. Note that the phase of the image antenna for a vertical antenna is the same as that of the installed antenna. Calculate the spacing d_z between the installed antenna and the image antenna.
6	b.	Treating this as a z-element z-directed antenna array, write an expression for the radiated electric field and power density for this antenna above the ground plane in terms of r, θ, ϕ.
6	c.	Calculate the angle/s of maximum radiation.
7	d.	Calculate the angle θ for first null of radiation.
7	e.	Calculate the gain of the antenna **including** the mutual impedance effects.

4.



a. The image antenna is sketched here. Center to center spacing d_z between the installed and the image antenna $= 0.8\lambda$

b. From Eq. (10) on p. 24 of the class Notes, the array factor for this 2 element ($N_z = 2$), $\hat{z}$ - directed antenna array can be written as

$$AF = e^{j\psi/2} \frac{\sin\psi}{\sin\psi/2} = 2\, e^{j\psi/2} \cos\left(\frac{\psi}{2}\right) \quad (1)$$

$$\text{where } \psi = \beta d_z \cos\theta = 1.6\pi\cos\theta \quad (2)$$

$\vec{E}_T = \vec{E}_0\, AF$ where for a "half wave" dipole, from p. 5 of the class Notes, Eq. 5-6

$$\vec{E}_0 = j\, 60 \frac{I_m}{r} F(\theta)\, e^{-j\beta r} = j\, 60 \frac{I_a}{r} F(\theta)\, e^{-j\beta r}$$

$$= j\, \frac{60}{r} \sqrt{2 P_{rad} R_a}\; F(\theta)\, e^{-j\beta r} \quad (3)$$

From Eq. (1) on p. 6 of Class Notes

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad (4)$$

Thus

$$E_T = \left[j\, \frac{60}{r} \sqrt{2 P_{rad} R_a}\; \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right] \cdot \left[2\, e^{j(0.8\pi\cos\theta)} \cos(0.8\pi\cos\theta) \right] \cdot e^{-j\beta r} \quad (5)$$

It is clear that E_T depends only on θ and is independent of angle ϕ, which is due to circular symmetry of this antenna.

c. From Eq. (5) on the previous page, for angles θ of max. radiation

$$0.8\pi \cos\theta = 0 \quad \text{i.e.} \quad \theta = 90^\circ$$ which is in the horizontal plane.

d. For the angle θ for first null of radiation, from Eq. (5)

$$0.8\pi \cos\theta_{FN} = 90^\circ = \frac{\pi}{2}$$

$$\theta_{FN} = \cos^{-1}\left(\frac{0.5}{0.8}\right) = 51.3^\circ$$

(Also the angle $\theta = 0^\circ$ for second null of radiation i.e. along the orientation of the antenna (z-axis) due to the term $\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}$.)

e. From p. 46 of class Notes for this z-directed colinear array

$$Z_1 = Z_{11} + Z_{12}\frac{I_2}{I_1} = Z_{11} + Z_{12}$$

($e^{j\alpha_z} \to 1$)

$$= (73 + j42) + (-2 - j5)$$

$$= 71 + j37$$

From Eq. 55-1 on p.33 of class Notes

$$D\Big|_{Array} = N^2 D_0 \frac{R_{A0}}{R_{A1}} = (2)^2 \times 1.64 \times \frac{73}{71}$$

$$= \boxed{6.744}$$

wj
ECE 5324
April 30, 2009

Name ____

Score:

Problem 1	_________	of a possible 30 points
Problem 2	_________	of a possible 20 points
Problem 3	_________	of a possible 30 points
Problem 4	_________	of a possible 30 points
Total	_________	of a possible 110 points