ECE 5324/6324
Final Exam. with Solutions

Name ______________________
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UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ANTENNA THEORY AND DESIGN

ECE 5324/6324

FINAL EXAMINATION

April 26, 2013

1. (25 points)

For a receiving antenna, the directivity D is given to be 500 at the reception frequency of 12,500 MHz. For this antenna, calculate

pts

6 a. the maximum effective area A_{em} at the incoming signal frequency.

7 b. the maximum received power P_r for an incoming signal power density of $S_{inc} = 1\ \mu W/m^2$.

6 c. the rms field strength E of the incoming signal.

6 d. the rms open circuit voltage developed across the antenna terminals given that R_A of the receiving antenna is 450 ohms.

1. a. For any antenna

$$D = \frac{4\pi A_{em}}{\lambda^2} \qquad (4\text{-}23 \text{ Text})$$

For a signal frequency of 12,500 MHz $\lambda = 2.4$ cm

$$A_{em} = \frac{D\lambda^2}{4\pi} = \frac{500 \times (2.4)^2}{4\pi} = 229.2 \text{ cm}^2 \Rightarrow 0.0229 \text{ m}^2$$

b. $P_r = S_{inc} A_{em} = 0.0229 \times 10^{-6} = 2.29 \times 10^{-8}$ W

c. $\frac{E_{rms}^2}{\eta} = 10^{-6} \Rightarrow E_{rms} = \sqrt{377 \times 10^{-3}} = 19.4 \frac{mV}{m}$ rms

$\rightarrow 27.44$ mV/m peak

d. From Eq. 23 of class Notes

$$V_{oc} = \sqrt{8 R_A S_{inc} A_{em}} = \sqrt{8 R_A P_r} = \sqrt{8 \times 450 \times 2.29 \times 10^{-8}}$$

$$= 90.8 \times 10^{-4} = 9.08 \text{ mV peak}$$

$\Rightarrow 6.42$ mV rms

2. (25 points)

A two-element Yagi antenna of identical length dipoles and inter-element separation of 0.1 λ is sketched in Fig. 1.

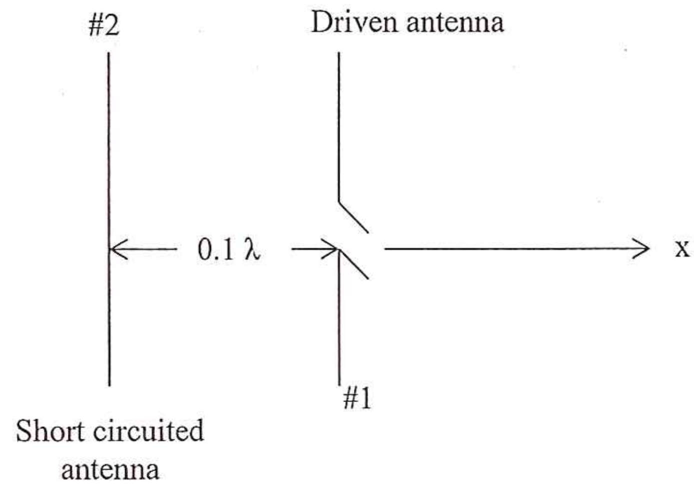

Fig. 1. A two-element Yagi antenna.

Given that the current $I_2/I_1 = 0.85\ e^{j50°}$ and the impedances $Z_{11} = Z_{22} = 50 - j100$ ohms,

pts

7 a. calculate the mutual impedance Z_{12} between the two antennas.

8 b. calculate the feed point impedance Z_1 for the driven antenna #1.

10 c. would this antenna system radiate more or less to the right side of antenna #1?

For 10 extra points:

d. calculate the **ratio** of the relative power densities on the right and left sides of the antenna along the x-axis.

2.

a. $V_1 = I_1 Z_{11} + I_2 Z_{12}$ for antenna 1 (1)

$0 = I_1 Z_{12} + I_2 Z_{22}$ for antenna 2 (2)

From Eq. (2)

$$\frac{I_2}{I_1} = -\frac{Z_{12}}{Z_{22}} = 0.85\, e^{j50^\circ}$$

$$Z_{12} = -0.85\, e^{j50^\circ} \underbrace{(50 - j100)}_{111.8\, e^{-j63.43}} = 95.03\, e^{j180^\circ} e^{-j13.43^\circ}$$

$$= 95.03\, e^{+j166.57^\circ} = -92.43 + j\,22.07\ \Omega$$

b. From Eq. (1)

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + \frac{I_2}{I_1} Z_{12} = (50 - j100) + \left(0.85\, e^{j50^\circ}\right)\left(95.03\, e^{j166.57^\circ}\right)$$

$$= (50 - j100) + 80.77\, e^{j216.5^\circ}$$

$$= \boxed{-14.92 - j\,148.04} = 148.77\, e^{-j95.76^\circ}$$

c. For the right side along the x-axis, the two vectors are

$E_{1R} \approx 0.85\, e^{j50^\circ} e^{-j36^\circ} e^{-j\beta x}$ and $E_{2R} \approx 1\, e^{j0} e^{-j\beta x}$

the total E_R vector for the right side

$$|E_R| = E_{1R} + E_{2R} = \Big(\underbrace{0.85\, e^{j14^\circ}}_{0.825 + j0.206} + 1\Big) = \left|(1.825 + j0.206)\right| = \boxed{1.836}$$

For the left side

$$|E_L| = |E_{1L} + E_{2L}| = \left|\left(0.85\, e^{j50^\circ} + 1\, e^{-j36^\circ}\right)\right|$$

$$= |0.546 + j0.651 + 0.809 - j\,0.588|$$

$$= |1.355 + j\,0.063| = \boxed{1.356}$$

The magnitude of the radiated E-field is larger to the right of antenna 1.

d. Ratio of power densities $= \dfrac{|E_R|^2}{|E_L|^2} = \left(\dfrac{1.836}{1.356}\right)^2 = 1.832$ (2.63 dB)

The radiated power density to the right of antenna 1 is larger by 83.2% than the radiated power density to the left of antenna 1.

3. (30 points)

A colinear antenna array of ten half-wavelength ($\lambda/2$) rectangular slots may be represented by a colinear array of complementary half-wave dipoles. Given that the center-to-center separation between this array along the z-axis is $d = \lambda$,

<u>pts</u>

7 a. write an expression for the array factor for this colinear $\hat{z}$-directed antenna array of slots in terms of angle θ relative to the z-axis.

8 b. calculate the phase shifts needed to get maximum/principal lobes of radiation for $\theta_o = \pm 45°$.

7 c. calculate the gain of the (slot) antenna array **neglecting** mutual impedance effects.

8 d. calculate the BWFN (beam width between first nulls) for the principal lobes.

3. a. From Eq. (10) on p. 25 of Class Notes, for a colinear $\hat{z}$ - directed antenna array

$$|AF| = \left| \frac{\sin(N\psi_z/2)}{\sin(\psi_z/2)} \right|$$

$\boxed{d_z = \lambda}$

$$\psi_z = \beta d_z \cos\theta - \alpha_z = 2\pi \cos\theta - \alpha_z$$

$$|AF| = \left| \frac{\sin(10\pi \cos\theta - 5\alpha_z)}{\sin(\pi\cos\theta - \frac{\alpha_z}{2})} \right|$$

b. For direction/s of maximum radiation

$$\frac{\psi_z}{2} = 0, \pm\pi, - -$$

$$\psi_z = 2\pi\cos\theta - \alpha_z = 0, \pm 2\pi, - -$$

for directions of maximum radiation $\theta_0 = \pm 45^\circ$

$$\alpha_z = 2\pi \cos 45^\circ, \; 2\pi\cos 45^\circ \mp 2\pi, - -$$

$$= \boxed{1.414\pi}, \; -0.586\pi$$

Note that both of these phases α_z are essentially the same since addition or substraction of 2π results in the same phase

Thus $\alpha_z = 254.5^\circ$ or -105.48°

c. $G = N \times 1.64 = \boxed{10.64} \Rightarrow 10.27^\circ$

d. BWFN is determined from the two directions of zero radiation for the major lobes

From Eq. (14) on p. 26 of Class Notes, for these directions

$$\psi = \pm \frac{2\pi}{N} = \pm \frac{\pi}{5}$$

$$2\pi\cos\theta - 1.414\pi = \pm\frac{\pi}{5} = \pm 0.2\pi$$

$$\cos\theta_{FN} = \frac{1.614\pi}{2\pi}; \; \frac{1.214\pi}{2\pi} = 0.807; \; 0.607$$

$$\theta_{FN} = 36.2^\circ, \; 52.63^\circ$$

$BWFN = \theta_{FN2} - \theta_{FN1} = \boxed{16.43^\circ}$

4. (30 points)

Design a rectangular aperture panel antenna that may be used as a base station antenna with a **half-power beamwidth** of 90° in the horizontal plane and 15° in the vertical plane for use at the cell phone frequency of 1900 MHz. Assuming a m = 1, cos (πX/L) type distribution,

pts

8 a. calculate the dimensions L_x and L_y for this rectangular aperture antenna along horizontal (x) and vertical (y) axes, respectively.

7 b. calculate the gain of the antenna in decibels.

8 c. what is the power received by this antenna from a user's cell phone given that $S_{inc} = 1\ \mu W/m^2$?

7 d. calculate the power amplification in dB required for a re-radiated power output of 5W.

Hint: This antenna may be considered similar to a pyramidal horn antenna with rectangular aperture.

4. A base station panel antenna is similar to a Pyramidal horn antenna where a Cosinusoidal $\cos\left(\frac{\pi X}{L}\right)$ type distribution of aperture fields is assumed for the x-axis and a uniform (non-varying) distribution of electric fields

From p. 77 of Class Notes

For the horizontal H plane or xz plane

$$\frac{80^\circ \lambda}{L_x} = 90^\circ \qquad L_x = \frac{8}{9}\lambda = \boxed{0.889\lambda} = \boxed{14.04\,cm}$$

For the vertical E plane or yz plane

$$\frac{53\lambda}{L_y} = 15^\circ \Rightarrow L_y = \frac{53}{15}\lambda = 3.53\lambda$$

For $f = 1900$ MHz, $\boxed{\lambda = 15.79\,cm} \Rightarrow \boxed{55.8\,cm}$

b. $$\text{Gain} = 0.51\,\frac{4\pi L_x L_y}{\lambda^2} = 0.51 \times 4\pi \times \frac{8}{9} \times 3.53$$

$$= 20.128 = 13\,dBi$$

c. $$P_r = S_{inc} A_e = S_{inc} \frac{\cancel{\lambda^2}}{\cancel{4\pi}} G\; 0.51 \frac{\cancel{4\pi} L_x L_y}{\cancel{\lambda^2}}$$

$$= 10^{-6} \times 0.51 \times 0.1404 \times 0.558 = 3.995 \times 10^{-8}\,W$$

d. Power amplification needed to boost the received power to 5W

$$= 10\log\left(\frac{5}{3.995 \times 10^{-8}}\right) = 80 + 0.97 = \boxed{80.97\,dB}$$

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Score:

Problem 1	_________	of a possible 25 points
Problem 2	_________	of a possible 25 points
Problem 3	_________	of a possible 30 points
Problem 4	_________	of a possible 30 points
Total	_________	of a possible 110 points