Midterm II with Solutions

Name ____
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UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ANTENNA THEORY AND DESIGN

ECE 5324/6324

MIDTERM II

March 29, 2013

1. (25 points)

A probe-fed rectangular microstrip patch antenna on a 0.7 cm thick substrate of $\varepsilon_r = 2.2$ is to be designed to operate at the WiMax frequency of 3670 MHz. Assuming that the width W of the patch is 0.9 L,

pts

5 a. find the patch length L at resonance.

7 b. find the input impedance if the patch is fed at the edge.

8 c. find the probe location for a 50Ω input impedance.

5 d. calculate the bandwidth B for this patch antenna.

1: a. From Eq. 11-1 on p. 468 of the Text $\lambda\Big|_{\substack{3670\\ \text{MHz}}} = \frac{30}{3.67} = 8.17$ Cm

$$L \simeq 0.49\,\lambda_d = 0.49\,\frac{\lambda}{\sqrt{\epsilon_r}} = 0.49\,\frac{8.17}{\sqrt{2.2}} = 2.7 \text{ Cm}$$

b. From Eq. 11-7 on p. 471 of the Text

$$Z_A = 90\,\frac{\epsilon_r^2}{\epsilon_r - 1}\left(\frac{L}{W}\right)^2 = 90\,\frac{4.84}{1.2}\times\left(\frac{1}{0.9}\right)^2 = 448.1\ \Omega$$

c. From Eq. (11-12) on p. 474 of the Text

$$Z_A(\Delta x_p) = 50 = \underset{\substack{\text{edge fed}\\ \text{patch}}}{Z_A\Big|}\cos^2\left(\frac{\pi\,\Delta x_p}{L}\right) = 448.1\cos^2\left(\frac{\pi\,\Delta x_p}{L}\right)$$

$$\cos\frac{\pi\,\Delta x_p}{L} = \sqrt{\frac{50}{448.1}} = 0.334 \Rightarrow \frac{\pi\,\Delta x_p}{L} = \cos^{-1}(0.334) = 70.49^\circ$$

$$\frac{\Delta x_p}{L} = 0.391 \rightarrow \Delta x_p = 0.391\,L = \boxed{1.057\text{ Cm}}$$

d. From Eq. (11-8) on p. 471 of the Text

$$B = 3.77\,\frac{\epsilon_r - 1}{\epsilon_r^2}\,\frac{W}{L}\,\frac{t}{\lambda} = 3.77\times\frac{1.2}{4.84}\times 0.9\times\frac{0.7}{8.17}$$

$$= 0.072 \quad \text{or} \quad 7.2\% \text{ of } 3670 \text{ MHz}$$

$$\boxed{B = 264.24 \text{ MHz}}$$

Note that if we do not need this large a bandwidth, we may use a lower value of t i.e. thickness of the substrate thus making the antenna even more compact

2. (25 points)

Calculate the feed point impedance of a “nominal quarter wave” monopole antenna $L/2 = (0.4781/2)\lambda$ that is placed at a distance of 0.3λ from a grounded “nominal quarter wave” monopole antenna $[L/2 = (0.4781/2)\lambda]$ for the geometry shown in Fig. 1.

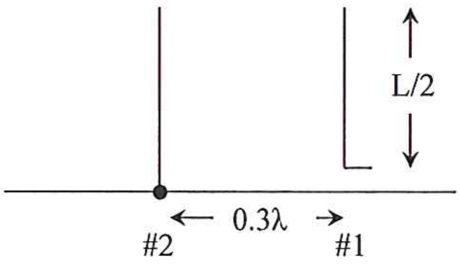

Fig. 1.

pts		
4	a.	Calculate the mutual impedance between antennas 1 and 2.
8	b.	Calculate the current I_2 induced in the grounded antenna #2 in terms of antenna current I_1.
8	c.	Calculate the feed point impedance Z_1 of antenna #1.
5	d.	For a transmitter power of 5 kW, calculate the current I_1 into the driven antenna 1.

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2. This problem is very similar to Example 21 of the class Notes except that the spacing between antennas 1 and 2 is $d = 0.3\lambda$ instead of 0.25λ in that example

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad (1)$$

$$0 = Z_{12} I_1 + Z_{22} I_2 \quad (2)$$

$$Z_{11} = \frac{73 + j42}{\underset{\text{monopole}\nearrow}{2}} = 36.5 + j21 = 42.1\, e^{j29.9^\circ}$$

From the graph on p.48 of Class Notes (enlarged version of Fig. 8-25 of Text)

$$Z_{12}\Big|_{d=0.3\lambda} = \frac{23 - j32}{\underset{\text{monopole}\nearrow}{2}} = \boxed{11.5 - j16} = 19.7\, e^{-j54.3^\circ}$$

From Eq.(2) above

$$\frac{I_2}{I_1} = \frac{-Z_{12}}{Z_{22}} = -\frac{Z_{12}}{Z_{11}} = -\frac{19.7\, e^{-j54.3^\circ}}{42.1\, e^{j29.9^\circ}}$$

$$= -0.47\, e^{-j84.2^\circ} = 0.47\, e^{j180^\circ} e^{-j84.2^\circ}$$

$$= \boxed{0.47\, e^{j95.8^\circ}} = -0.047 + j0.467$$

From Eq. (1)

$$Z_1 = \frac{V_1}{I_1} = Z_{11} + \frac{Z_{12} I_2}{I_1} = \frac{(73 + j42)}{2} + 19.7\, e^{-j54.3^\circ} \times 0.47\, e^{j95.8^\circ}$$

$$= 36.5 + j21 + 9.26\, e^{j41.5^\circ}$$

$$= 36.5 + j21 + (6.9 + j6.1)$$

$$= 43.4 + j27.1 = 51.2\, e^{j32^\circ}$$

d. $\frac{1}{2} I_1^2 R_{a1} = 5000$ $I_1\Big|_{\text{peak}} = \sqrt{\frac{10000}{43.4}} = 15.18$ A peak

→ 10.73 A rms

3. (25 points)

Design a two-reactance network of arrangement shown in Fig. 2 to **conjugate** match an antenna of impedance $Z_A = 40 - j100$ ohms to a source of impedance $Z_S = 80 + j100$ ohms.

pts

15 a. Calculate the two possible values of reactances jX_{sh} that may be used for the circuit.

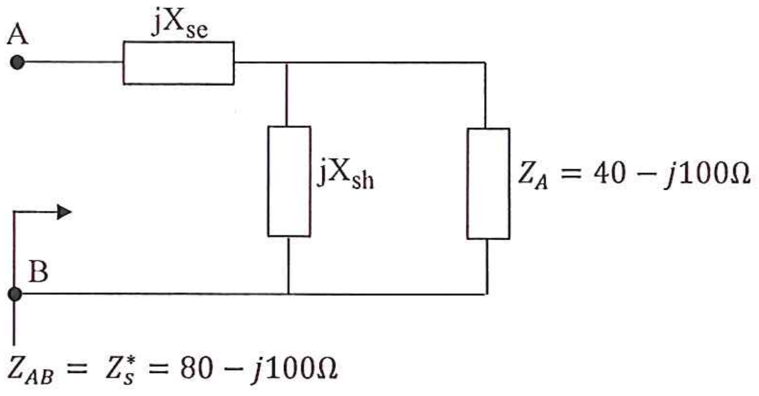

Fig. 2.

10 b. To determine X_{se}, take the **smaller** of the two possible values of X_{sh}.

3. For this problem we follow a procedure very similar to that on pp. 57 and 58 of class Notes

$$Z_{AB} = \frac{jX_{sh}(40 - j100)}{40 + j(X_{sh}-100)} + jX_{se}$$

$$= \frac{jX_{sh}[40 - j(X_{sh}-100)][40 - j100]}{(40)^2 + (X_{sh}-100)^2} + jX_{se}$$

$$= \frac{[100X_{sh} + j40X_{sh}][40 - j(X_{sh}-100)]}{(40)^2 + (X_{sh}-100)^2} + jX_{se} \qquad (1)$$

$$= 80 - j100 \ \Omega$$

Equating real parts on both sides of Eq. (1)

$$80[(40)^2 + (X_{sh}-100)^2] = \cancel{4000X_{sh}} + 40X_{sh}(X_{sh}\cancel{-100})$$

$$= 40X_{sh}^2$$

$$(40)^2 + (X_{sh}-150)^2 = 0.5X_{sh}^2 \qquad (2)$$

$$X_{sh}^2 - 200X_{sh} + 11600 = 0.5X_{sh}^2$$

$$0.5X_{sh}^2 - 200X_{sh} + 11600 = 0$$

$$X_{sh} = \frac{200 \pm \sqrt{\underset{40{,}000}{(200)^2} - 4\times \underset{23200}{\cancel{11600 \times 0.5}}}}{1} = 200 \pm \sqrt{16{,}800}$$

$$= 200 \pm 129.6 = 329.6 \ ; \ 70.38$$

We take the small value $X_{sh} = 70.38\ \Omega$

For this value, upon equating the imaginary parts of Eq. 1, we get

$$\frac{(40)^2 \overset{70.38}{\cancel{X_{sh}}} - 100\overset{70.38}{\cancel{X_{sh}}}(\underset{-29.62}{\cancel{X_{sh}-100}})}{0.5X_{sh}^2} = \frac{208{,}481 + 112{,}608}{2476.67} = (129.6 + X_{se})$$

from Eq. (2) →

$$= -100$$

$$\boxed{X_{se} = -229.6\ \Omega} \leftarrow \text{Answer}$$

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Score:

Problem 1 _________ of a possible 25 points

Problem 2 _________ of a possible 25 points

Problem 3 _________ of a possible 25 points

Total _________ of a possible 75 points