# UNIVERSITY OF UTAH ELECTRICAL \& COMPUTER ENGINEERING DEPARTMENT 

18. Calculate the feed point impedances and radiation patterns for the following 3-element antenna arrays:
a. Side-by-side parallel dipole antennas each of length $\mathrm{L}=0.4781 \lambda$ (nominal half-wave dipoles); inter-element distance $\mathrm{d} / \lambda=0.40$; progressive phase difference $\alpha=-144^{\circ}$ between adjacent antenna elements (use the enlarged version of Text Fig. 8-25 (p. 48 of the Class Notes) for mutual impedances).

Note that this is an end fire antenna array since $\alpha=-\beta \mathrm{d}$.
b. Colinear dipole antennas each of length $\mathrm{L}=0.4781 \lambda$ with an end-to-end separation $\mathrm{s}=0.20 \lambda ; \alpha=0^{\circ}$, i.e., all antennas are fed in phase [use enlarged version of Text Fig. 8-26 (p. 49 of the Class Notes)].
19. Using the NEC Code, calculate the feed point impedances for the various elements of the 3 -element antenna arrays of the above Problem 18. Compare your results with those obtained in Problem 18 and explain differences, if any.
20. For a transmitter power of 5 KW , calculate the feed point currents for each of the antenna elements in Problem 18 parts $\mathrm{a}, \mathrm{b}$. Calculate the directivity for each of the antenna arrangements.

From Eq. $55-\mathrm{a}$ on page 37 of Class Notes,

$$
\mathrm{D}=\mathrm{N}^{2} \mathrm{D}_{\mathrm{o}} \frac{\mathrm{R}_{\mathrm{Ao}}}{\sum_{\mathrm{i}=1}^{3} \mathrm{R}_{\mathrm{Ai}}}
$$

For $\theta=90^{\circ}$ plane, calculate the maximum power density for a total radiated power of 5 KW at a distance of 20 miles.
21. Using NEC Code or otherwise, calculate the radiation pattern for a 4-element array stretched along the x -axis with an inter-element spacing $\mathrm{d} / \lambda=0.40$ for:
a. An ordinary end fire array $\left(\alpha=-\beta \mathrm{d}=-2 \pi \times 0.40=-144^{\circ}\right)$.
b. An increased directivity end-fire array (Hanson-Woodyard Array; see p. 285 Text).

Assume half wave dipoles for the elements of the array and obtain the radiation patterns for $\theta=90^{\circ}$ xy plane. Give a table comparing the following quantities for each of the above arrays:

1. Beam width between first nulls.
2. Half power beam width.
3. Directivity.
4. Levels of first side lobes in dB below principal lobe.

Include mutual impedance effects for calculating the directivity of the above two antennas.
22. A certain antenna installation using a transmitting frequency of 1.0 MHz uses two vertical towers of height 200 and 240 feet with a separation of 650 feet between the towers installed along the East-West line. The antenna driving point currents are:

|  | East Tower | West Tower |
| :--- | :---: | :---: |
|  | Magnitude | 1 |$:$| 0.60 |
| :---: |
| Phase |

Calculate the radiation pattern of the antenna in the horizontal plane $\theta=90^{\circ}$ and for $\theta=30^{\circ}$ and $60^{\circ}$. Calculate the maximum field intensity at a distance of 100 km in the horizontal plane and the gain of the antenna. For the total radiated power of 25 kW , calculate the magnitudes of currents fed to the two antennas.
23. Repeat Problem 22 assuming that the West Tower is not fed, but is grounded instead.
24. For the antenna of Problem 22, calculate the field intensity at a distance of 1800 km . Since the fields at that distance are primarily due to the reflection from the ionosphere, knowledge of the effective height of the ionosphere is necessary. For this problem, this may be taken to be 350 km above ground. Include in your calculations the fact that the earth is round with a radius of 3960 miles. Assume a $3-\mathrm{dB}$ loss in reflection from the ionosphere.

## Yagi-Uda Array

25. Calculate the radiation pattern of a Yagi-Uda Antenna of four elements of specifications as follows (see the sixth row of the attached Table 8.1):

$$
\begin{aligned}
& \text { Spacing }=0.15 \lambda \\
& \text { Reflector length }=0.505 \lambda \\
& \text { Driver length }=0.476 \lambda \\
& \text { Three directors of length }=0.456 \lambda
\end{aligned}
$$

Use the NEC Code and see if the numbers given in the following are correct (from attached Table 8.1):

$$
\begin{aligned}
& \text { Gain }=10.0 \mathrm{~dB} \\
& \text { Front-to-back ratio }=13.1 \mathrm{~dB} \\
& \text { Input impedance }=9.6+\mathrm{j} 13.0 \text { ohms }
\end{aligned}
$$

For H-plane ( $\theta=90^{\circ}$, xy plane $)$ :

$$
\mathrm{HPBW}=76^{\circ}
$$

$$
\mathrm{SLL}=-8.9 \mathrm{~dB}
$$

## Short Wave Antennas

26. Design a short-wave ( $\mathrm{f}=30 \mathrm{MHz}$ ) antenna system of $\lambda / 2$ horizontal dipoles above ground in order to obtain a single lobe of maximum radiation at an angle $\phi_{o}=24^{\circ}$ relative to ground appropriate for communication with a receiving site 2000 km away. What is the gain of this antenna?

Hint: Use a procedure similar to Ex. 21 on pp. 62-65 of Class Notes. Start with a $\lambda / 2$ horizontal dipole spaced appropriately above ground and compute its radiation pattern using the NEC Code. You will find that this antenna system gives several lobes. Add on more $\lambda / 2$ dipoles that are appropriately spaced to achieve the objective of obtaining a single lobe of maximum radiation.

## Corner Reflector Antennas

27. Obtain the radiation patterns, gains, and feed point impedances of the following corner reflector antennas with a half-wave dipole.
a. $\quad \beta=180^{\circ}$ flat reflector for $\mathrm{S}=0.10 \lambda$.
b. $\beta=90^{\circ}$ corner reflector for $\mathrm{S}=0.10 \lambda$.
c. $\beta=90^{\circ}$ corner reflector for $\mathrm{S}=1.0 \lambda$.

For each of the above antennas, plot the radiation patterns for $\theta=90^{\circ}$ and $\theta=$ $30^{\circ}$ planes (for $0<\phi<360^{\circ}$ ).

Calculate the gains of the above three antennas and compare these with the values given in the graphs on p. 63 of Class Notes..

Compare the calculated feed point impedances of the driven antennas in parts $a, b, c$ of this problem with the values given in graphs on p. 64 of the Class Notes.
28. A $180^{\circ}$ corner reflector antenna with two $\lambda / 2$-driven elements in phase is sketched in Fig. 1. The distance for each of the elements to the reflector is $0.15 \lambda$ and the center-to-center spacing is $0.70 \lambda$.


Calculate the gain of this antenna. Calculate the HPBW in the H-plane $\left(\theta=90^{\circ}\right.$ or xy plane) for the above antenna.

Calculate the feed point impedance/s for the two dipoles including the mutual impedance effects.

## Traveling Wave Antennas

29. Using Eq. 7-7 of the text, calculate and plot the angle of maximum radiation $\theta_{\mathrm{m}}$ as a function of $\mathrm{L} / \lambda$ varying from 1.0 to 10.0 in steps of 0.4 .

Compare your results with the plot shown in Fig. 7-4 of the text.

## TV Antenna

30. Four $\lambda / 2$ vertical dipoles are mounted on a vertical mast with a center-to-center spacing $\mathrm{d}=0.70 \lambda$. If the antennas are fed in phase and the distance from the center point of the lowest element to ground is $0.70 \lambda$, write the expression for the radiated power density of the antenna as a function of $\theta$ and $\varphi$. Calculate and plot the power density as a function of angle $\theta$ for $2^{\circ} \leq \theta \leq 90^{\circ}$ in steps of $\Delta \theta=2^{\circ}$. Calculate the gain of this antenna neglecting mutual impedance effects.
Table 8.1 Characteristics of equally spaced Uda-Yagi antennas (Source: W. L. Stutzman and G. A. Thiele. (1981), Antenna Theory and Design Wiley, New York. Reproduced by permission of John Wiley \& Sons Inc.)

| Number of elements. | Spacing (wave lengths) | Element lengths (wavelengths) |  |  | Gain (dB) | Front-toback ratio (dB) | $\begin{gathered} \text { Input } \\ \text { impedance } \\ \text { (ohm) } \end{gathered}$ | $H$-plane |  | $E$-plane |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Reflector | Driver | Director |  |  |  | $\begin{aligned} & \mathrm{HP}_{\mathrm{H}} \\ & (\mathrm{deg}) \end{aligned}$ | $\underset{(\mathrm{dB})}{\mathrm{SLL}_{\mathrm{H}}}$ | $\underset{{ }_{(d e g}}{\mathrm{HP}_{\mathrm{E}}}$ | $\begin{aligned} & \operatorname{SLL}_{\mathrm{E}} \\ & (\mathrm{~dB}) \end{aligned}$ |
| 3 | 0.25 | 0.479 | 0.453 | 0.451 | 9.4 | 5.6 | $22.3+$ j15.0 | 84 | -11.0 | 66 | -34.5 |
| 4 | 0.15 | 0.486 | 0.459 | 0.453 | 9.7 | 8.2 | $36.7+$ j9.6 | 84 | -11.6 | 66 | -22.8 |
| 4 | 0.20 | 0.503 | 0.474 | 0.463 | 9.3 | 7.5 | $5.6+\mathrm{j} 20.7$ | 64 | -5.2 | 54 | -25.4 |
| 4 | 0.25 | 0.486 | 0.463 | 0.456 | 10.4 | 6.0 | $10.3+$ j23.5 | 60 | - 5.8 | 52 | -15.8 |
| 4 | 0.30 | 0.475 | 0.453 | 0.446 | 10.7 | 5.2 | $25.8+$ j23.2 | 64 | -7.3 | 56 | -18.5 |
| 5 | 0.15 | 0.505 | 0.476 | 0.456 | 10.0 | 13.1 | $9.6+$ j 13.0 | 76 | -8.9 | 62 | -232 |
| 5 | 0.20 | 0.486 | 0.462 | 0.449 | 11.0 | 9.4 | $18.4+$ j 17.6 | 68 | -8.4 | 58 | -18.7 |
| 5 | 0.25 | 0.477 | 0.451 | 0.442 | 11.0 | 7.4 | $53.3+\mathrm{j} 6.2$ | 66 | -8.1 | 58 | -19.1 |
| 5 | 0.30 0.20 | 0.482 0.482 | 0.459 0.456 | 0.451 | 9.3 | 2.9 | $19.3+\mathrm{j} 39.4$ | 42 | -3.3 | 40 | -9.5 |
| 6 | 0.20 0.25 | 0.482 0.484 | 0.456 0.459 | 0.437 0.446 | 11.2 | 9.2 | $51.3-\mathrm{jl} 1.9$ | 68 | -9.0 | 58 | -20.0 |
| 6 | 0.25 0.30 | 0.484 0.472 | 0.459 0.449 | 0.446 0.437 | 11.9 | 9.4 | $23.2+\mathrm{j} 21.0$ | 56 | -7.1 | 50 | -13.8 |
| 6 | 0.30 | 0.472 0.489 | 0.449 | 0.437 | 11.6 | 6.7 | $61.2+j 7.7$ | 56 | -7.4 | 52 | -14.8 |
| 7 | 0.20 | 0.489 | 0.463 | 0.444 | 11.8 | 12.6 | $20.6+\mathrm{j} 16.8$ | 58 | -7.4 | 52 | -14.1 |
| 7 | 0.25 | 0.477 | 0.454 | 0.434 | 12.0 | 8.7 | $57.2+\mathrm{j} 1.9$ | 58 | -8.1 | 52 | -15.4 |
| 7 | 0.30 | 0.475 | 0.455 | 0.439 | 127 | 8.7 | $35.9+$ j21.7 | S0 | -7.3 | 46 | -12.6 |

Conductor diameter $=0.005 ;$
HP $=$ half-power beamwidth
SLL $=$ Side-lobe level

